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INTERACTIVE FUZZY GOAL PROGRAMMING FOR ORDER ALLOCATION WITH MULTIPLE CRITERIA AND MULTIPLE CONSTRAINT LEVELS

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Abstract

This paper presents a multiple criteria and multiple constraint-level linear programming (MC^2 LP) model for order quantity allocation under price breaks in a multi-decision-maker environment. Order allocation is inherently a multi-objective problem influenced by several conflicting criteria. In practice, suppliers often offer price discounts, and multiple decision makers (DMs) are involved in the decision process. To capture the uncertainty in DMs' opinions, the model incorporates multiple constraint levels on demand. The proposed approach considers three objective functions: minimizing purchasing cost, the number of late deliveries, and the number of rejects. An interactive fuzzy goal programming procedure, extended with a modified two-phase method, is developed to obtain Pareto-optimal solutions. A numerical example is provided to illustrate the application of the proposed model, and a comparative analysis with existing methods is conducted. The results demonstrate that the MC^2 LP model is effective for order allocation under price discount schemes in multi-DM settings, and that the proposed method successfully identifies Pareto-optimal solutions.

Keywords: Order allocation; Pareto-optimal solution; Quantity discounts; Multi-objective programming; Multi-criteria and multi-constraint level linear programming; MC²LP

INTRODUCTION[1]

Outsourcing has become increasingly prevalent as firms strive to focus on value-added activities and core competencies to enhance their competitive advantage and improve customer satisfaction in today's dynamic and competitive market. A key concern in outsourcing decisions is the allocation of business across suppliers. Buyers aim to maintain flexibility and avoid over-dependence on any single source, while suppliers, particularly small ones, worry that sudden disruptions in purchasing may threaten their financial viability. Therefore, making effective outsourcing decisions requires buyers to exercise considerable judgment in allocating order quantities among available suppliers to satisfy strategic objectives.

Order allocation constitutes a multi-criteria decision-making (MCDM) problem. Suppliers are often evaluated based on a combination of technical, engineering, and logistical capabilities. Traditional evaluation criteria—such as quality, delivery, and price—remain among the most frequently cited factors in supplier assessment (Arikan, 2013). Additionally, real-world order allocation problems frequently involve price discounts, offering buyers opportunities to meet cost objectives. Accordingly, developing effective strategies to manage such pricing dynamics is essential for procurement professionals.

Many researchers approach order quantity allocation using single-objective techniques, such as linear programming (LP) or mixed-integer programming (MIP), where cost is typically the primary objective and other criteria are modeled as constraints. However, such formulations—featuring a single objective and a single constraint level—often fall short in capturing the complexity of real-world scenarios. In practice, order allocation decisions frequently involve input from multiple departments or stakeholders within a firm (Dyer and Forman, 1992). Cross-functional sourcing teams, for instance, bring together individuals with diverse expertise to address procurement challenges. These teams tend to outperform individuals by leveraging broader skills, knowledge, and perspectives (Johnson et al., 2011). However, the diversity of opinions, particularly regarding parameters like total demand, introduces uncertainty into the decision-making process. Consequently, order allocation is better framed as a group decision-making problem involving multiple criteria and multiple constraint levels.

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[1] DM – decision maker
$\mathrm{FMC^2LP}$ - fuzzy multiple criteria and multiple constraint level linear programming
LP – Linear programming
MCLP - multiple criteria linear programming
$\mathrm{MC^2LP}$ - multiple criteria and multiple constraint-level linear programming
MIP – Mixed integer programming

Traditional LP and multi-criteria linear programming (MCLP) models generally assume a single decision-maker and a single level of resource availability. In contrast, multiple criteria and multiple constraint-level linear programming ($\mathrm{MC^2LP}$) extends the MCLP framework by accommodating both conflicting criteria and varying levels of resource constraints. These multiple constraint levels—reflecting the discrete opinions of different decision-makers—are critical when the decision process involves a group. Thus, the group MCLP problem should consider multiple resource levels to reflect the practical realities of decision-making in organizations. The $\mathrm{MC^2LP}$ model provides a more realistic framework than single-level models (Liu and Shi, 1994). One example is production system design, where financial, production, and marketing managers may define different constraint levels (Shi and Liu, 1997). Accordingly, $\mathrm{MC^2LP}$ is more appropriate than MCLP for solving order allocation problems involving multiple decision-makers with distinct views on resource availability. The model accommodates multiple evaluation criteria—such as price, delivery performance, and product quality—while incorporating diverse constraints defined by stakeholders including finance, quality, and procurement managers.

Although mathematical programming techniques are commonly used to tackle multi-objective problems, solving $\mathrm{MC^2LP}$ models is challenging due to the involvement of multiple decision-makers with differing preferences. While the $\mathrm{MC^2}$-simplex method can address such problems, it often requires significant computational effort due to the need to explore a large number of pivot operations. This complexity reduces the method's practicality for large-scale problems. Furthermore, it is often difficult to identify which of the potential solutions lie on the surface or vertex of the polyhedron, complicating the final decision-making process. Decision-makers may hesitate to select a specific solution from among many candidates, especially when trade-offs are not clearly defined.

To better handle such uncertainties, fuzzy MCLP techniques have been employed to solve $\mathrm{MC^2LP}$ problems (Liu and Shi, 1994). However, many existing fuzzy MCLP solution methods generate fuzzy-efficient rather than Pareto-optimal solutions, which may limit their practical utility (Jimenez and Bilbao, 2009). Notably, Pareto-optimal solutions form a subset of weakly Pareto-optimal solutions, and selecting a final compromise solution often requires the integration of subjective judgment with quantitative analysis. Besides fuzzy approaches, goal programming (GP) and interactive methods are commonly used to guide the decision-making process toward a compromise solution.

In light of these considerations, this study proposes an integrated procedure that combines fuzzy set theory, interactive decision-making, and goal programming to identify Pareto-optimal solutions for the $\mathrm{MC^2LP}$ order allocation problem. The model accounts for quantity discounts and incorporates multiple constraint levels to reflect the perspectives of various decision-makers. Furthermore, a modified two-phase method is developed within the proposed framework to enhance solution quality and efficiency.

The remainder of this paper is structured as follows. Section 2 reviews the literature on $\mathrm{MC^2LP}$ and order allocation methodologies. Section 3 introduces the $\mathrm{MC^2LP}$ model and outlines the proposed solution methodology. Section 4 provides a numerical example and presents a comparative analysis of solutions. Finally, Section 5 concludes the study and offers recommendations for future research.

LITERATURE REVIEW

The literature review will focus on the methods to solve the order allocation and the $\mathrm{MC^2LP}$ problems.

Solution methods for the order allocation problem

The order allocation problem has been widely studied through the lenses of mathematical programming, multi-criteria decision-making, fuzzy logic, group decision-making (GDM), and quantity discount strategies. This section reviews the evolution of solution methods, beginning with mathematical and hybrid programming approaches and extending to models that incorporate fuzziness, multiple objectives, discount considerations, and group-based decision-making.

Numerous researchers have employed mathematical programming models to optimize order allocation based on cost and supplier performance metrics. For instance, Ghodsypour and O'Brien (2001) proposed a mixed-integer nonlinear programming (MINLP) model to minimize total logistics cost under budget, quality, and service constraints. Talluri and Narasimhan (2003) introduced a max-min approach to evaluate supplier efficiency scores and allocate order quantities accordingly. Faez et al. (2009) formulated a mixed-integer programming (MIP) model to concurrently select suppliers and determine order quantities under demand and capacity limitations. Bohner and Minner (2017) addressed supplier failure risks and quantity discounts using a mixed-integer linear programming (MILP) model. These models primarily focus on single-objective optimization, where one criterion is treated as the objective and others as constraints.

To overcome the limitations of single-objective frameworks, several studies have adopted multi-objective and goal programming techniques. Ghodsypour and O'Brien (2001), for example, proposed a hybrid model integrating the Analytic Hierarchy Process (AHP) with MINLP for order allocation. Similarly, Kumar et al. (2007) developed a hybrid AHP-goal programming model for vendor selection. Choudhary and Shankar (2014) presented a goal programming model addressing inventory lot-sizing, supplier selection, and carrier determination simultaneously. However, these approaches typically lack mechanisms to account for uncertainty and participatory decision-making.

To address this, a subset of the literature incorporates fuzziness and multiple objectives to better reflect real-world procurement scenarios. Jadidi et al. (2008) combined TOPSIS with fuzzy multi-objective MILP to determine order quantities with price breaks. Wu et al. (2010) introduced a fuzzy multi-objective linear programming (FMOLP) model using a possibility-based solution approach. Amid et al. (2011) integrated AHP-derived weights into a fuzzy multi-objective model for supplier evaluation, while Kumar et al. (2017) treated demand as a fuzzy variable in a fuzzy AHP-weighted FMOLP model. Although these models recognize uncertainty and conflicting objectives, they often neglect scenarios involving multiple constraint levels, as found in group decision-making contexts.

More recent research has incorporated sustainability and Pareto optimization. Cheraghalipour and Farsad (2018) developed a bi-objective MILP model minimizing total cost while maximizing supplier sustainability scores, using a Revised Multi-Choice Goal Programming (RMCGP) solution method. Mohammed et al. (2019) integrated fuzzy AHP and fuzzy TOPSIS to evaluate suppliers across traditional, environmental, and social criteria. They solved the resulting fuzzy multi-objective optimization model using ε-constraint and LP-metrics methods to generate Pareto-optimal solutions, followed by TOPSIS-based selection. Moheb-Alizadeh and Handfield (2019) proposed a comprehensive multi-objective MILP model considering multiple periods, products, and transportation modes, filtering Pareto solutions via DEA-based super efficiency scores. Mirzaee et al. (2022) incorporated environmental regulations, such as cap-and-trade mechanisms, into a robust optimization model for green supplier selection and order allocation in closed-loop supply chains.

Quantity discount strategies have also attracted scholarly attention. Wadhwa and Ravindran (2007) developed a multi-objective model that incorporates price, lead time, and quality with discount structures. Wang and Yang (2009) employed AHP and fuzzy compromise programming to address quantity discounts. Amid et al. (2009) introduced a fuzzy multi-objective model considering price breaks. Cebi and Otay (2016) applied the fuzzy MULTIMOORA method for supplier evaluation and utilized a max–min fuzzy goal programming model encompassing discounts, lead time, capacity, and demand. Gupta et al. (2016) proposed a possibilistic programming approach integrating fuzzy multi-objective integer programming and AHP for sustainable vendor selection under price breaks and fuzzy constraints.

Despite these advances, limited research has addressed group decision-making in the context of order allocation. Chou and Chang (2008) employed a fuzzy multi-attribute rating method incorporating group decision-making for supplier selection. Razmi et al. (2009) combined fuzzy TOPSIS with fuzzy MIP to allocate orders based on group input. Zouggari and Benyoucef (2012) utilized a simulation-based fuzzy TOPSIS approach for supplier selection and order allocation within a group decision-making setting. Although these studies incorporate fuzzy group decision-making, they fall short of modeling scenarios where multiple decision-makers specify divergent constraint levels.

In practice, procurement decisions are frequently made by committees of decision-makers (DMs), each with distinct preferences, evaluations, and risk perceptions. While decision aids and heuristics are commonly used to navigate uncertainty, they often fail to address the ambiguity and subjectivity inherent in group decision-making. Divergent opinions among DMs can be modeled as heterogeneous constraint levels; however, current mathematical formulations rarely incorporate this complexity. No reviewed model to date has integrated multiple constraint levels derived from different DMs within a unified order allocation framework. This gap highlights a critical research opportunity: the development of advanced models that integrate fuzziness, multi-objective optimization, and group decision-making with heterogeneous constraints reflective of real-world organizational settings.

MC^{2}LP solution methods

There is a paucity of literature applying multiple-criteria and multiple-constraint-level linear programming (MC^{2}LP) to practical decision-making problems. A few notable exceptions include He et al. (2010) and Shanmugapriya (2012), who employed MC^{2}LP in data mining, particularly for classification problems in credit card scoring. Zhong et al. (2013) extended the application of MC^{2}LP to oil field development. In addition, Chen et al. (2013) provided a comprehensive review of the MC^{2}LP framework. However, to the best of the author’s knowledge, no existing studies have applied the MC^{2}LP approach to the order allocation problem—especially one that simultaneously accounts for divergent decision-maker (DM) opinions and quantity-based price discounts within a unified modeling framework.

Seiford and Yu (1979) initially proposed the MC^2-simplex method to solve MC^{2}LP problems, extending the traditional simplex algorithm of linear programming to handle multiple criteria and multiple constraint levels. However, this method suffers from significant computational complexity, as it requires considerable time to identify all potential efficient solutions, which substantially limits its practical use in large-scale problems. Furthermore, it poses difficulties for DMs, who may struggle to select a single preferred solution from a set of potentially large alternatives.

To address these limitations, Shi and Liu (1993) and Liu and Shi (1994) developed a procedure that integrates the MC²-simplex method with fuzzy linear programming (FLP), thereby enabling the generation of fuzzy-efficient solutions. While these fuzzy approaches offer greater flexibility in representing uncertainty and imprecision in DM preferences, they have a critical drawback: once a goal has been fully satisfied (i.e., the membership grade reaches 1), the corresponding solution may no longer be Pareto-optimal (Pal et al., 2003).

To bridge these gaps in the existing literature, this study develops a novel MC²LP model tailored for the order allocation problem under multiple conflicting objectives. The model simultaneously considers minimizing purchasing costs, the number of defective items, and the number of late deliveries, all subject to real-world constraints such as buyer demand, supplier capacity, and quantity-based price discounts. Moreover, recognizing the diversity of opinions among multiple DMs and the inherent conflict among objectives, this study proposes an interactive fuzzy decision-making procedure. The proposed approach combines a modified two-phase method with fuzzy logic to guide the search for a Pareto-optimal solution that balances conflicting goals while incorporating heterogeneous constraint levels specified by different DMs.

MODEL DEVELOPMENT

A general multiple-criteria linear programming (MCLP) problem can be formulated as

Max Cx

s.t. $Ax \leq d$, $x \geq 0$ (1)

where $x \in R^n$ represents the decision variables, $C \in R^{K \times n}$ is the matrix of objective function coefficients, $A \in R^{m \times n}$ is the constraint matrix, and $d \in R^m$ represents a single resource availability vector.

To incorporate multiple constraint levels from different decision-makers (DMs), an $m \times p$ matrix $D = [D_1, \ldots, D_p]$ is introduced, where each column D_w represents a constraint vector corresponding to the opinion or estimation of DM DMw. The convex hull generated by the columns of D, denoted by $\mathrm{H}(D)$, is defined as:

$$\mathrm{H}(D) = \{d : d = \sum_{w=1}^{p} \gamma_w D_w, \gamma_w \geq 0, \sum_{w=1}^{p} \gamma_w = 1\}$$

An MCLP becomes a multiple-criteria, multiple-constraint level linear programming (MC²LP) problem when the constraint is relaxed to $Ax \leq d$ for some $\mathrm{d} \in \mathrm{H}(D)$. It is important to note that the MCLP is a special case of the $\mathrm{MC^2LP}$ when D consists of a single column, i.e., $\mathrm{P} = 1$.

In the $\mathrm{MC^2LP}$ formulation, a solution x is considered feasible if the constraint $Ax \leq d$ is satisfied for a $\mathrm{d} \in \mathrm{H}(D)$, meaning that Ax lies within the convex combination of the constraint vectors D_w. For a given weight vector $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_p) > 0$, with $\sum_{w=1}^{p} \gamma_w = 1$, Liu and Shi (1994) demonstrated that a solution x^* is a potential solution to the $\mathrm{MC^2LP}$ if and only if it is an efficient solution to the following parametric linear program:

Max Cx

s.t. $Ax \leq \gamma D$, $x \geq 0$ (2)

where γD denotes the linear combination of constraint vectors determined by the weights γ, reflecting the aggregated preference or opinion among multiple DMs.

Components of the $\mathrm{MC^2LP}$ Order Allocation Model

In the context of order allocation under price discounts and group decision-making, the $\mathrm{MC^2LP}$ model includes the following components:

Decision Variables

- x_{ij}: Quantity allocated to supplier i at price level j
- Y_i: Binary selection variable; $Y_i = 1$ if supplier i is selected, and 0 otherwise

Parameters

- n: Total number of available suppliers
- p_{ij}: Unit price offered by supplier i at price level j
- v_i: Number of price levels offered by supplier i
- D_w: Demand estimation provided by decision-maker w
- Q_{ij}: Maximum allowable quantity that can be allocated to supplier i at price level j
- l_i: Percentage of late deliveries from supplier i
- r_i: Percentage of defective items received from supplier i
- c_i: Maximum capacity available from supplier i

This model structure facilitates the simultaneous consideration of multiple objectives—such as cost minimization, quality improvement, and on-time delivery performance—under the complex constraint environment shaped by multiple decision-makers' judgments and quantity-based price discount structures. It provides a more realistic and adaptable framework for solving multi-criteria order allocation problems in uncertain and multi-actor settings.

MC²LP Order Allocation Model Formulation

Based on the purchasing policy assumptions regarding price, delivery reliability, and product quality, the order allocation problem can be formulated as a multiple-criteria, multiple-constraint level linear programming ($\mathrm{MC^2LP}$) model. This formulation accommodates the existence of multiple constraint levels in demand estimation, reflecting the views of different decision-makers (DMs). Assuming at least two DMs are involved in the process, the $\mathrm{MC^2LP}$ model for the order allocation problem is defined as follows:

Objective Functions

Minimize:

$G_1 = \sum_{i=1}^{n} \sum_{j=1}^{\nu_i} p_{ij} x_{ij}$ (f 1)

$G_2 = \sum_{i=1}^{n} \sum_{j=1}^{\nu_i} l_i x_{ij}$ (f 2)

$G_3 = \sum_{i=1}^{n} \sum_{j=1}^{\nu_i} r_i x_{ij}$ (f 3)

s.t.

$\sum_{j=1}^{\nu_i} x_{ij} \leq c_i, \ \forall i$ (f 4)

$\sum_{i=1}^{n} \sum_{j=1}^{\nu_i} x_{ij} = \gamma_1 D_1 + \cdots + \gamma_p D_p$ (f 5)

$Q_{i,j-1} Y_{ij} \leq x_{ij} \leq Q_{ij} Y_{ij}, \forall i, \forall j$ (f 6)

$\sum_{j=1}^{\nu_i} Y_{ij} \leq 1, \ \forall i$ (f 7)

$x_{ij} \geq 0, \ \forall i, \ \forall j$ (f 8)

$Y_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases} \ \forall i, \ \forall j$ (f 9)

$\sum_{w=1}^{p} \gamma_w = 1, \gamma_w \geq 0$ (f10)

Model Description

- Objective Functions (f1–f3): The model includes three conflicting objectives:
  - (f1) Minimizing total purchasing cost.
  - (f2) Minimizing the number of late deliveries.
  - (f3) Minimizing the number of defective items (rejects).
- Constraint (f4): Ensures that the total quantity allocated to each supplier does not exceed their available capacity.
- Constraint (f5): Represents the incorporation of multiple constraint levels, allowing each DM to express a different opinion on the total demand. The aggregated demand is modeled as a convex combination of the individual DMs' estimates.
- Constraint (f6): Enforces the valid range of order quantities corresponding to the selected price break for each supplier. The quantity x_{ij} must lie between the lower and upper bounds defined by the price break thresholds $Q_{i,j-1}$ and $Q_{i,j}$, respectively, if supplier i is chosen at price level j.

- Constraint (f7): Limits each supplier to be selected at no more than one price break level.
- Constraint (f8): Prevents negative order allocations.
- Constraint (f9): Specifies the binary nature of the supplier–price-level selection decision variable Y_{ij}.
- Constraint (f10): Ensures that the DMs' preference weights γ_w form a valid convex combination.

This model is formulated as a mixed-integer linear programming (MILP) problem due to the inclusion of binary variables Y_{ij}, enabling clearer and more interpretable solutions for the decision-makers. The use of the MC²LP framework allows for systematic incorporation of multiple, and potentially conflicting, demand opinions from different DMs, while simultaneously addressing cost, delivery, and quality considerations in supplier selection and order allocation.

Fuzzy Solution Approach

In fuzzy programming, both the objective function and multiple constraints are treated as fuzzy sets. Let G and C denote the fuzzy sets representing the objectives and the constraints, respectively. A "decision" in a fuzzy program is defined as one that simultaneously satisfies the objective functions and constraints to an acceptable degree. According to Bellman and Zadeh (1970), the fuzzy decision set D is defined as the intersection of G and C, with the corresponding membership function given by:

$$\mu_D(\mathrm{x}) = \min\{\mu_G(\mathrm{x}), \mu_C(\mathrm{x})\}$$

The upper bound U_k and lower bound L_k of the decision maker's (DM's) acceptability for each objective *k* represent, respectively, the worst (maximum) and best (minimum) possible values of the *k*-th objective. These bounds can be determined by solving the corresponding single-objective optimization problems (Lai and Hwang, 1994).

Let:

$$G_k = \{x: L_k \le cx \le U_k\}$$

Then, the linear membership function associated with minimizing the objective G_k is defined as:

$$\mu_{G_k}(x) = \begin{cases} 1, & \text{if } cx \le L_k \\ \dfrac{U_k - cx}{U_k - L_k}, & \text{if } L_k \le cx \le U_k\,, \\ 0, & \text{if } cx \ge U_k \end{cases} \qquad \forall k$$

For multiple objectives, the overall fuzzy objective set is:

$\mathrm{G} = \cap_{k=1}^{K} G_k$

According to the min-operator principle (Bellman and Zadeh, 1970), the fuzzy decision is thus defined as:

$$\max \mu_D(x) = \max_x \min\{\mu_G(x), \mu_C(x)\} = \min\{\mu_G(x^*), \mu_C(x^*)\}$$

Following the concept of membership functions and the fuzzy decision operator (Zimmermann, 1978), the Multi-Criteria Linear Programming (MCLP) problem (1) can be reformulated as:

Maximize β

s.t.

$\beta \leq \mu_{G_k}(x),\ \forall k$

$x \in X = \{Ax \leq d, x \geq 0\}$

$\beta \in [0,1]$ (3)

Here, the auxiliary variable β represents the overall satisfaction level or degree of compromise. Based on this formulation, the optimal solution of the following fuzzy multiple criteria and multiple constraint-level linear program ($\mathrm{FMC^2LP}$) is considered a feasible solution for the original $\mathrm{MC^2LP}$ problem (Liu and Shi, 1994):

Maximize β

s.t.

$\beta \leq \mu_{G_k}(x),\ \forall k$

$$Ax \leq \sum_{w=1}^{p} \gamma_w D_w$$

$$\sum_{w=1}^{p} \gamma_w = 1$$

$\beta \in [0,1],\ x \geq 0, \gamma_w \geq \varepsilon > 0$ for given ε (4)

This fuzzy approach systematically reduces the complexity of the multi-objective model by converting the $\mathrm{MC^2LP}$ problem into a standard linear program. If the $\mathrm{FMC^2LP}$ has a unique optimal solution, then this solution is fuzzy-efficient (see Definition 1). Otherwise, while not all optimal solutions may be fuzzy-efficient, at least one of them will be (Werners, 1987).

Definition 1 (Werners, 1987).
A solution $x^\circ \in X$ is fuzzy-efficient with respect to the $\mathrm{FMC^2LP}$ if and only if there is no other solution $y \in X$ such that $\mu_{G_k}(y) \geq \mu_{G_k}(x^\circ)$ for all k and $\mu_{G_s}(y) > \mu_{G_s}(x^\circ)$ for at least one index s.

Definition 2 (Werners, 1987).
A solution $x^\circ \in X$ is Pareto-optimal to the $\mathrm{MC^2LP}$ if and only if there is no other $y \in X$ such that $G_k(y) \leq G_k(x^\circ)$ for all k, and $G_s(y) < G_s(x^\circ)$ for at least one s.

A proposed approach

Numerous algorithms have been developed to generate fuzzy-efficient solutions to transformed models in multi-criteria linear programming contexts. Notable examples include the works of Lee and Li (1993), Sakawa (1993), Guu and Wu (1999), and Arikan and Gungor (2007). These approaches often employ fuzzy membership functions to transform multi-objective problems into single-objective formulations. However, as discussed by Li and Lai (2000), a fuzzy-efficient solution derived from such transformations (e.g., model (4)) may not be Pareto-optimal in the context of a Multi-Criteria Linear Programming (MCLP) problem (see Definition 2).

Building on the method proposed by Guu and Wu (1999), Jiménez and Bilbao (2009) introduced a Fuzzy Multi-Objective Linear Programming (FMOLP) model to obtain a solution that is both fuzzy-efficient and Pareto-optimal. Nonetheless, in their model, the value of each fuzzy goal is determined subjectively by the decision maker (DM), rather than being based on the feasible region of the objective functions. This subjectivity introduces the risk of infeasibility if the DM-specified aspiration levels fall outside the attainable domain of the problem.

To mitigate this issue, the present study assumes that DMs are unable to express precise aspiration levels for the objectives. Consequently, an interactive two-phase approach is proposed to identify Pareto-optimal solutions without requiring the DM to specify exact desired values. This approach modifies conventional fuzzy goal programming techniques and integrates them into a robust MCLP framework with multiple constraint levels.

We propose solving the following optimization problem:

Max $\sum_{k=1}^{K} d_k$

s.t.

$G_k(x) + d_k = G_k(x^*), \qquad \forall k = 1,2,\dots,K$

$$Ax \le \sum_{w=1}^{p} \gamma_w D_w$$

$$\sum_{w=1}^{p} \gamma_w = 1$$

$\gamma_w \ge \varepsilon > 0$, $\forall w$

$x \in X$, $x \ge 0$, $d_k \ge 0$ (5)

Here, $G_k(x^*)$ represents the value of the *k*-th objective function in the solution x^* obtained from model (4), which corresponds to the fuzzy-efficient solution. The variables d_k measure the degree of overachievement (or improvement) in the *k*-th goal, similar to negative deviation variables in conventional Goal Programming (GP). The coefficients γ_w are weights assigned to each constraint level D_w, satisfying the convex combination condition and a lower-bound ε to avoid degenerate allocations.

This two-phase method ensures that the final solution is Pareto-optimal by first identifying a fuzzy-efficient point, and then maximizing the aggregate overachievement of goals based on feasible bounds, without imposing infeasible aspirations. Moreover, it accommodates the uncertain and imprecise nature of group decision-making by avoiding reliance on exact numerical inputs from the DMs.

Lemma 1.

If x^o is an optimal solution for problem (5), then x^o is a Pareto-optimal solution for problem (2).

Proof.

Assume, for the sake of contradiction, that the optimal solution x^o of problem (5) is not a Pareto-optimal solution of problem (2). Then, there exists another feasible solution y to problem (2) such that:

$G_k(y) \leq G_k(x^o),\ \ \forall k = 1,2,\dots,K,$

and

$G_s(y) < G_s(x^o)$, for at least one $s \in \{1,\dots,K\}$.

Given that the membership functions $\mu_{G_k}(x)$ are strictly monotonically decreasing with respect to $G_k(x)$, we have:

$\mu_{G_k}(x^o) \leq \mu_{G_k}(y),\ \ \forall k,$

which implies that y is also feasible for problem (5).

From the definition of d_k in problem (5), we have:

$$G_k(x^o) + d_k(x^o) = G_k(x^*), \qquad \forall k$$

Then, the objective value of problem (5) at x^o is:

$\sum_{k=1}^{K} d_k(x^o) = \sum_{k=1}^{K}[G_k(x^*) - G_k(x^o)].$

Since $G_s(y) < G_s(x^o)$ and $G_k(y) \leq G_k(x^o)$ for all other k, it follows that:

$\sum_{k=1}^{K}[G_k(x^*) - G_k(x^o)] = \sum_{k,k\neq s} G_k(x^*) - G_k(x^o) - G_s(x^o) \ < \sum_{k,k\neq s} G_k(x^*) - G_k(y) - G_s(y) =$
$\sum_{k=1}^{K}[G_k(x^*) - G_k(y)] = \sum_{k=1}^{K} d_k(y).$

In other words, the following inequality holds: $\sum_{k=1}^{K} d_k\,(x^o) < \sum_{k=1}^{K} d_k\,(y)$. This contradicts the assumption that x^o is an optimal solution of problem (5). Hence, the contradiction implies that x^o must be a Pareto-optimal solution to problem (2). □

Interactive Improvement Process

In practice, determining precise aspiration levels for multiple conflicting goals is often difficult for decision makers (DMs). As a result, the solutions obtained from the fuzzy goal programming models may not always be optimal in the Pareto sense unless a saturation point—where the membership value equals 1—is achieved for all objectives. Except in such ideal cases, a fuzzy-efficient solution to the $\mathrm{FMC^2LP}$ (Fuzzy Multi-Criteria and Multi-Constraint Linear Programming) problem may still fall short of Pareto optimality in the $\mathrm{MC^2LP}$ problem.

To address this, an interactive improvement procedure is recommended. This approach allows the DM to progressively adjust preferences throughout the solution process, improving dissatisfying objectives without requiring precise aspiration levels. Each objective function is represented as a fuzzy number characterized by a linear membership function that reflects the DM's preferences. These membership functions are updated dynamically in response to evolving tolerances for each objective.

The interactive procedure involves the following steps:

1. Initial Solution: Solve model (4) using current membership functions to obtain a fuzzy-efficient solution x^*.
2. Construct Model (5): Use x^* as input to model (5) and solve for the Pareto-optimal solution x^o.
3. Check Satisfaction: Compare $G_k(x^o)$ with the upper tolerance limit U_k for each objective:
    - If $G_k(x^o) < U_k$, update U_k with this new tighter bound and revise the membership function accordingly.
    - Otherwise, retain the current membership function.
4. Iterate: With updated membership functions, resolve models (4) and (5).
5. Repeat until:
    - The DM is satisfied with the solution, or
    - No significant improvement can be made to the objective values.

This iterative process narrows the feasible region by eliminating dissatisfying solutions and focusing on regions of higher satisfaction. It converges when no further meaningful improvements can be made—guaranteeing termination of the algorithm. However, it is important to note that improvements in one membership function may come at the expense of deterioration in another, due to trade-offs inherent in multi-objective optimization (Sakawa et al., 1987).

The described procedure is practical and can be implemented using widely available mathematical programming software such as LINGO, GAMS, or MATLAB.

Solution Procedure

The following steps summarize the proposed interactive fuzzy goal programming solution procedure for the $\mathrm{MC^2LP}$ (Multi-Criteria and Multi-Constraint Level Linear Programming) problem. This approach integrates fuzzy set theory and multi-phase optimization to iteratively achieve a Pareto-optimal solution while incorporating the decision maker's (DM's) preferences:

Step 1: Formulate the $\mathrm{MC^2LP}$ problem as presented in model (2), defining all objective functions and constraints.

Step 2: For each objective function $G_k(x)$, determine its individual maximum and minimum values by optimizing it separately under the problem's constraints. These bounds are necessary to construct the membership functions.

Step 3: Using the upper and lower bounds obtained in Step 2, construct linear membership functions $G_k(x)$ for each objective function to represent the DM's preferences.

Step 4: Solve the max–min fuzzy model (4) (Phase I) to identify a fuzzy-efficient solution.

- If the solution is unique, proceed to Step 8.
- If multiple fuzzy-efficient solutions exist, continue to Phase II (Step 5).

Step 5: Add the optimal value from Phase I as a constraint and solve model (5) to improve upon the previous solution and generate a Pareto-optimal solution.

Step 6: Present the solution to the decision maker (DM). If the DM is satisfied with the result, proceed to Step 8. Otherwise, continue to Step 7.

Step 7: For each objective function $G_k(x^o)$:

- If $G_k(x^o) < U_k$, update U_k as the new upper bound, reconstruct the corresponding membership function, and return to Step 3.
- Otherwise, retain the current membership function and return to Step 3.

Step 8: Stop. The final solution is either a unique fuzzy-efficient solution or a Pareto-optimal solution obtained through iterative refinement in collaboration with the DM.

This iterative and interactive procedure ensures progressive improvement of the solution through preference adjustment and model re-solving. It balances conflicting objectives while reducing the feasible region based on updated tolerances, ultimately yielding a DM-satisfactory and Pareto-optimal solution.

```
# Step 1: Define the MC²LP model
define_model()

# Step 2: Compute individual min and max for each objective G_k
bounds = {}
```

```
for k in objectives:
    bounds[k] = {
        "min": optimize(G_k, sense="minimize"),
        "max": optimize(G_k, sense="maximize")
    }

# Step 3: Construct linear membership functions μ_Gk(x)
mu_functions = construct_membership_functions(bounds)

# Phase I: Solve max-min model (4)
solution_phase1 = solve_max_min_model(mu_functions)

if solution_phase1.is_unique:
    final_solution = solution_phase1
    stop()
else:
    while True:
        # Phase II: Solve model (5)
        solution_phase2 = solve_model_5(solution_phase1)

        # Step 6: Interaction with the decision maker
        if DM_is_satisfied(solution_phase2):
            final_solution = solution_phase2
            break
        else:
            # Step 7: Update membership functions
            for k in objectives:
                if solution_phase2.objective_values[k] < bounds[k]["max"]:
                    bounds[k]["max"] = update_upper_bound(k)
            mu_functions = construct_membership_functions(bounds)
            solution_phase1 = solve_max_min_model(mu_functions)

# Output the final solution
return final_solution
```

NUMERICAL EXAMPLE AND PERFORMANCE ANALYSIS

Model formulation and basic data for the example problem

This section demonstrates the application of the proposed Multi-Criteria and Multi-Constraint Level Linear Programming (MC²LP) model to an order allocation problem. The aim is to illustrate the procedure for solving the problem using the developed methodology.

Table 1 presents the relevant data for three potential suppliers, including their respective price levels (G_1), delivery performance (G_2), quality capability (G_3), and capacity limitations. For illustrative purposes, it is assumed that there are three candidate suppliers for a single product. The decision-makers (DMs) have specified two estimated levels of demand: 800 units and 1200 units.

The proposed procedure is implemented using the LINGO v.14 optimization software. The solution process involves the following steps:

Step 1: Formulate the $\mathrm{MC^2LP}$ model for the example problem (see Appendix for full model formulation).

Step 2: Solve model (2) to obtain the minimum and maximum values for each objective function. The results are summarized in Table 2.

Step 3: Construct linear membership functions for each objective function based on the computed minimum and maximum values from Step 2.

Step 4: In Phase I, the focus is on transforming model (2) into model (4), which incorporates the developed membership functions to address the multi-objective nature of the problem.

Max β

Subject to:

$$\beta \leq \frac{12600 - (10x_{11} + 9.5x_{12} + 9x_{13} + 12x_{21} + 11.5x_{22} + 11x_{23} + 8x_{31} + 7.5x_{32} + 7x_{33})}{12600 - 5600}$$

$$\beta \leq \frac{220 - 0.1(x_{11} + x_{12} + x_{13}) - 0.2(x_{21} + x_{22} + x_{23}) - 0.15(x_{31} + x_{32} + x_{33})}{220 - 80}$$

$$\beta \leq \frac{228 - 0.2(x_{11} + x_{12} + x_{13}) - 0.1(x_{21} + x_{22} + x_{23}) - 0.15(x_{31} + x_{32} + x_{33})}{228 - 80}$$

$$\gamma_1 \geq 0.3$$

$$\gamma_2 \geq 0.3$$

$$\beta \geq 0$$

and (A.1) – (A.19).

The optimal solution obtained is $x_{11}^* = 146$, $x_{21}^* = 114$, and $x_{33}^* = 660$. The optimal value is $\gamma_1 = 0.7$, $\gamma_2 = 0.3$, $\beta^* = 0.597$, $G_1(x^*) = 7448$, $G_2(x^*) = 136.4$, $G_3(x^*) = 139.6$, and $\mu_1(x^*) =$

0.736, $\mu_2(x^*) = 0.5971$, $\mu_3(x^*) = 0.5973$. According to Definition 1, this solution is fuzzy efficient, although its Pareto-optimality remains uncertain. Therefore, we proceed to Phase II.

Phase II: Interactive Improvement via Deviation Variables

We now solve the following model to improve the fuzzy efficient solution:

Max $d_1 + d_2 + d_3$

s.t.

$10x_{11} + 9.5x_{12} + 9x_{13} + 12x_{21} + 11.5x_{22} + 11x_{23} + 8x_{31} + 7.5x_{32} + 7x_{33} + d_1 = 7448$

$0.1(x_{11} + x_{12} + x_{13}) + 0.2(x_{21} + x_{22} + x_{23}) + 0.15(x_{31} + x_{32} + x_{33}) + d_2 = 136.4$

$0.2(x_{11} + x_{12} + x_{13}) + 0.1(x_{21} + x_{22} + x_{23}) + 0.15(x_{31} + x_{32} + x_{33}) + d_3 = 139.6$

$\gamma_1 \geq 0.3$, $\gamma_2 \geq 0.3$, d_1, d_2, $d_3 \geq 0$

and (A.1) – (A.19)

The improved solution is: $x_{11}^o = 32$, $x_{33}^o = 888$, $G_1(x^o) = 6536$, $G_2(x^o) = 136.4$, and $G_3(x^o) = 139.6$. This solution demonstrates improvement over the previous step, as: $G_1(x^o) < G_1(x^*)$, $G_2(x^o) = G_2(x^*)$, and $G_3(x^o) = G_3(x^*)$.

Step 6. If the decision-maker is not satisfied with the current outcome, the membership function for the unsatisfactory objective (in this case, G_1) is updated using the current value as the new upper bound.

Step 7. The new upper bound for G_1 becomes 6536, while membership functions for G_2 and G_3 remain unchanged. Re-solving the updated model yields the same optimal solution: $x_{11}^o = 32$, $x_{33}^o = 888$ and $G_1(x^o) = 6536$, $G_2(x^o) = 136.4$, and $G_3(x^o) = 139.6$. No further improvement is observed, and since the solution is now both fuzzy efficient and Pareto-optimal, the algorithm terminates. Table 3 summarizes the iterative results.

Sensitivity to Decision Maker Preferences

Table 4 summarizes the optimal solutions under various preference scenarios. In Case 1, 83% of the total order is allocated to Supplier 3 due to superior price performance. The preference weights $\gamma_1 = 1.0$, $\gamma_2 = 0$ indicate that the first decision maker's preferences dominate.

As γ_2 increases (Cases 2–4), the importance of satisfying the second DM grows. Consequently, total demand increases, and Supplier 3 continues to receive the majority of the order due to consistently favorable performance. However, allocations to Supplier 2 decline.

This analysis demonstrates the model's flexibility in adjusting to varying DM preferences. Trade-offs among suppliers can be visualized graphically (Figure 1), aiding the DM in understanding solution behavior.

Comparative Performance with Alternative Methods

Table 5 presents results from several comparative cases:

- Cases 2–4: 10% increases in p_{ij}, l_i, and r_i, respectively
- Cases 5–7: 10% joint increases in combinations of the above parameters

The following insights are derived:

- The membership function structure reflects the DMs' imprecise preferences by using objective lower and upper bounds.
- The max–min and fuzzy programming approaches effectively quantify satisfaction levels, facilitating informed decision-making.
- While the max–min solutions yield acceptable satisfaction levels, they may not guarantee Pareto optimality.
- The proposed method, via a two-phase fuzzy interactive procedure, consistently yields lower G_1 values than Liu and Shi's (1994) method, while maintaining identical values for G_2 and G_3.
- Iterative updates to unsatisfactory membership functions, via reduced tolerance intervals, progressively refine the solution space, enabling convergence to Pareto-optimal outcomes.

In summary, the proposed method proves superior in generating efficient, preference-sensitive, and Pareto-optimal solutions under fuzzy multi-objective constraints.

Performance analysis

This section evaluates the performance of the solution results obtained using the proposed method in comparison with Liu and Shi's (1994) fuzzy programming approach, as reported in Table 5. The weak potential solution derived from the max-min method is excluded from this comparison due to its inferior performance across the evaluated objectives.

To identify the most effective method, each compromise solution for the objectives was compared against its corresponding ideal value. The closeness of each compromise solution to its ideal counterpart was measured using a family of distance functions proposed by El-Wahed and Lee (2006). This measure quantifies the aggregate distance between the compromise and ideal solutions across all objectives, as follows:

$$D_p(w,K) = [\sum_{k=1}^{K} w_k^p (1-d_k)^{1/p}]$$

where d_k denotes the degree of closeness between the compromise and ideal values for the kkkth objective, w_k is the normalized relative importance (or weight) of the kth objective

($\sum_{k=1}^{K} w_k = 1$), and p is a parameter representing the type of distance function, with $1 \leq p \leq \infty$. For specific values of p, the distance functions simplify to:

- $D_1(w_k, K) = 1 - \sum_{k=1}^{K} w_k d_k$
- $D_2(w_k, K) = [\sum_{k=1}^{K} w_k^2 (1 - d_k)^2]^{1/2}$
- $D_\infty(w_k, K) = \max_k \{w_k(1 - d_k)\}$

In a minimization context, d_k is computed as the ratio of the optimal solution of G_k to the preferred (ideal) value of G_k. The preferred method is the one that yields the shortest distance to the ideal point under the respective distance metric.

Table 6 presents the comparison results for Liu and Shi's (1994) fuzzy programming method and the proposed approach under different weighting schemes for the objective functions. The results show that the proposed method consistently yields lower values of D_1 and D_2, indicating a closer proximity to the ideal solution across a variety of preference weight settings. While the D_∞ values are equal for both methods regardless of the weights, the superiority of the proposed approach in terms of overall closeness is evident under the more commonly used D_1 and D_2 measures.

These findings confirm that the proposed interactive two-phase programming method provides a more desirable compromise solution than Liu and Shi's (1994) fuzzy programming approach.

From a managerial perspective, the proposed Multi-Criteria and Multi-Constraint Level Linear Programming (MC^2LP) model offers practical advantages for solving order allocation problems. In real-world applications, multiple decision-makers (DMs) and quantity discount schemes frequently complicate procurement decisions. The proposed formulation accommodates multiple conflicting objectives, integrates the preferences of multiple DMs, and accounts for price discounts simultaneously.

Furthermore, the approach maintains flexibility by allowing adjustments in the number of objectives and constraints to suit dynamic organizational policies and market conditions. As demonstrated in the numerical example, sensitivity analysis based on the DMs' opinion weights is feasible within the proposed framework. This adaptability empowers firms to tailor procurement decisions to diverse and evolving strategic needs.

Importantly, the proposed method guarantees that the solution is not only fuzzy efficient but also Pareto-optimal, providing strong support for its implementation as a robust decision-making tool in multi-objective procurement environments.

CONCLUSION

Most purchasing decisions involve multiple decision-makers (DMs), culminating in a group decision formed by aggregating individual judgments. Unlike previous studies that address the multi-choice linear programming (MCLP) order allocation problem under a single constraint level, this study proposes a Multi-Choice Multi-Constraint Linear Programming (MC²LP) model. The proposed model offers a more comprehensive framework for optimal order allocation by incorporating organizational objectives, multiple constraint levels derived from DMs' opinions, and price discount considerations. As such, it more accurately reflects real-world applications.

A key contribution of this study lies in the solution approach developed for the MC²LP problem. Specifically, a novel procedure integrating fuzzy sets, interactive techniques, and a two-phase method is designed to identify Pareto-optimal solutions. By iteratively comparing solution values and the tolerance limits of membership functions, the membership function is dynamically updated to efficiently obtain a satisficing solution that balances the preferences of multiple DMs.

Although the proposed method is more complex than traditional max–min and fuzzy linear programming approaches, it enables better interaction with DMs and more effectively captures group preferences. Additionally, it requires fewer iterations to reach a preferred compromise solution. One limitation of the approach is that it may fail to identify an appropriate Pareto-optimal solution when the feasible space is non-convex.

The main contribution of this study is the development of an effective approach for solving the MC²LP model, which is applicable to various order allocation problems. Future research could explore constraints related to strategic policies, such as minimum order quantity percentages assigned to suppliers. Further extensions may include applying the approach to multi-product scenarios or using alternative types of membership functions for the objective function. Ultimately, the proposed method holds promise for practical application in real-world MC²LP problems, including transportation, inventory, and production planning.

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APPENDICES

Min $G_1 = 10x_{11} + 9.5x_{12} + 9x_{13} + 12x_{21} + 11.5x_{22} + 11x_{23} + 8x_{31} + 7.5x_{32} + 7x_{33}$

Min $G_2 = 0.1(x_{11} + x_{12} + x_{13}) + 0.2(x_{21} + x_{22} + x_{23}) + 0.15(x_{31} + x_{32} + x_{33})$

Min $G_3 = 0.2(x_{11} + x_{12} + x_{13}) + 0.1(x_{21} + x_{22} + x_{23}) + 0.15(x_{31} + x_{32} + x_{33})$

Subject to:

$x_{11} + x_{12} + x_{13} \leq 960$ (A.1)

$x_{21} + x_{22} + x_{23} \leq 800$ (A.2)

$x_{31} + x_{32} + x_{33} \leq 1000$ (A.3)

$x_{11} + x_{12} + x_{13} + x_{21} + x_{22} + x_{23} + x_{31} + x_{32} + x_{33} \leq 800\gamma_1 + 1200\gamma_2$ (A.4)

$0 \leq x_{11} \leq 239Y_{11}$ (A.5)

$$240Y_{12} \leq x_{12} \leq 479Y_{12} \tag{A.6}$$

$$480Y_{13} \leq x_{13} \leq 960Y_{13} \tag{A.7}$$

$$0 \leq x_{21} \leq 179Y_{21} \tag{A.8}$$

$$180Y_{22} \leq x_{22} \leq 593Y_{22} \tag{A.9}$$

$$594Y_{23} \leq x_{23} \leq 800Y_{23} \tag{A.10}$$

$$0 \leq x_{31} \leq 329Y_{31} \tag{A.11}$$

$$330Y_{32} \leq x_{32} \leq 659Y_{32} \tag{A.12}$$

$$660Y_{33} \leq x_{33} \leq 1000Y_{33} \tag{A.13}$$

$$Y_{11} + Y_{12} + Y_{13} \leq 1 \tag{A.14}$$

$$Y_{21} + Y_{22} + Y_{23} \leq 1 \tag{A.15}$$

$$Y_{31} + Y_{32} + Y_{33} \leq 1 \tag{A.16}$$

$$x_{ij} \geq 0,\ i, j = 1, 2, 3;\ x_{ij} = integer \tag{A.17}$$

$$Y_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases};\ i, j = 1, 2, 3 \tag{A.18}$$

$$\gamma_1 + \gamma_2 = 1 \tag{A.19}$$

$$\gamma_1 \geq 0,\ \gamma_2 \geq 0 \tag{A.20}$$

Table 1. Numerical example: Data

Supplier	*Quantity level*	*Price ($)*	*% of late delivery*	*% of rejects*	*Capacity constraint*
S1	$Q < 239$	10	0.1	0.2	960
	$240 \leq Q < 479$	9.5			
	$480 \leq Q$	9			
S2	$Q < 179$	12	0.2	0.1	800
	$180 \leq Q < 593$	11.5			
	$594 \leq Q$	11			
S3	$Q < 329$	8	0.15	0.15	1000
	$330 \leq Q < 659$	7.5			
	$660 \leq Q$	7			

Table 2. Numerical example: Upper and lower bounds

Objective function	*Maximum value*	*Minimum value*
G_1	12600	5600
G_2	220	80
G_3	228	80

Table 3. Numerical example: Solutions

Iteration	1		2	
Phase	I	II	I	II
S_1	x_{11} =146	x_{11} =32	x_{11} =32	x_{11} =32
S_2	x_{21} =114	$x_{2\bullet}$ =0	$x_{2\bullet}$ =0	$x_{2\bullet}$ =0
S_3	x_{33} =660	x_{33} =888	x_{33} =888	x_{33} =888
G_1	7448	6536	6536	6536
G_2	136.4	136.4	136.4	136.4
G_3	139.6	139.6	139.6	139.6
μ_1	0.736	0.866	0.866	0.866
μ_2	0.597	0.597	0.597	0.597
μ_3	0.597	0.597	0.597	0.597

Table 4. Numerical example: Case analysis

Case	(γ_1,γ_2)	S_1	S_2	S_3	G_1	G_2	G_3
1. (ε =0)	(1,0)	x_{11}=81	x_{21}=59	x_{33}=660	6138	118.9	121.1
2. (ε =0.3)	(0.7,0.3)	x_{11}=32	$x_{2\bullet}$ =0	x_{33}=888	6536	136.4	139.6
3. (ε =0.5)	(0.5,0.5)	x_{11}=38	$x_{2\bullet}$ =0	x_{33}=962	7114	148.1	151.9

Table 5. Comparison of solutions for different models

Case	Method	S_1	S_2	S_3	G_1	G_2	G_3	μ_1	μ_2	μ_3	(γ_1, γ_2)
1	Max-min	x_{11}=81	x_{21}=59	x_{33}=660	6138	118.9	121.1	0.923	0.7221	0.7223	(1.0, 0)
	Fuzzy programming (Liu and Shi, 1994)	x_{11}=146	x_{21}=114	x_{33}=660	7448	136.4	139.6	0.736	0.5971	0.5973	(0.7, 0.3)
	Proposed approach	x_{11}=32	$x_{2.}$=0	x_{33}=888	6536	136.4	139.6	0.866	0.5971	0.5973	(0.7, 0.3)
2^a	Max-min	x_{11}=81	x_{21}=59	x_{33}=660	6219	118.9	121.1	0.992	0.7221	0.7223	(1.0, 0)
	Fuzzy programming (Liu and Shi, 1994)	x_{11}=146	x_{21}=114	x_{33}=660	8192	136.4	139.6	0.736	0.5971	0.5973	(0.7, 0.3)
	Proposed approach	x_{11}=32	$x_{2.}$=0	x_{33}=888	7189	136.4	139.6	0.866	0.5971	0.5973	(0.7, 0.3)
3^b	Max-mi	x_{11}=81	x_{21}=59	x_{33}=660	6138	130.8	121.1	0.923	0.7221	0.7223	(1.0, 0)
	Fuzzy programming (Liu and Shi, 1994)	x_{11}=146	x_{21}=114	x_{33}=660	7448	150.0	139.6	0.736	0.5971	0.5973	(0.7, 0.3)
	Proposed approach	x_{11}=32	$x_{2.}$=0	x_{33}=888	6536	150.0	139.6	0.866	0.5971	0.5973	(0.7, 0.3)
4^c	Max-min	x_{11}=81	x_{21}=59	x_{33}=660	6138	118.9	133.2	0.923	0.7221	0.7223	(1.0, 0)
	Fuzzy programming (Liu and Shi, 1994)	x_{11}=146	x_{21}=114	x_{33}=660	7448	136.4	153.5	0.736	0.5971	0.5973	(0.7, 0.3)
	Proposed approach	x_{11}=32	$x_{2.}$=0	x_{33}=888	6536	136.4	153.5	0.866	0.5971	0.5973	(0.7, 0.3)
5^d	Max-min	x_{11}=81	x_{21}=59	x_{33}=660	6751	130.8	121.1	0.923	0.7221	0.7223	(1.0, 0)
	Fuzzy programming (Liu and Shi, 1994)	x_{11}=146	x_{21}=114	x_{33}=660	8192	150.0	139.6	0.736	0.5971	0.5973	(0.7, 0.3)
	Proposed approach	x_{11}=32	$x_{2.}$=0	x_{33}=888	7189	150.0	139.6	0.866	0.5971	0.5973	(0.7, 0.3)

	Max-min	x_{11}=81	x_{21}=59	x_{33}=660	6751	118.9	133.2	0.923	0.7221	0.7223	(1.0, 0)
6^e	Fuzzy programming (Liu and Shi, 1994)	x_{11}=146	x_{21}=114	x_{33}=660	8192	136.4	153.5	0.736	0.5971	0.5973	(0.7, 0.3)
	Proposed approach	x_{11}=32	$x_{2.}$=0	x_{33}=888	7189	136.4	153.5	0.866	0.5971	0.5973	(0.7, 0.3)
	Max-min	x_{11}=81	x_{21}=59	x_{33}=660	6138	130.8	133.2	0.923	0.7221	0.7223	(1.0, 0)
7^f	Fuzzy programming (Liu and Shi, 1994)	x_{11}=146	x_{21}=114	x_{33}=660	7448	150.0	153.5	0.736	0.5971	0.5973	(0.7, 0.3)
	Proposed approach	x_{11}=32	$x_{2.}$=0	x_{33}=888	6536	150.0	153.5	0.866	0.5971	0.5973	(0.7, 0.3)

*Where 2^a: Δp_{ij}=10%; 3^b: Δl_i=10%; 4^c: Δr_i=10%; 5^d: Δp_{ij}=10% and Δl_i=10%; 6^e: Δp_{ij}=10% and Δr_i=10%; 7^f: Δl_i=10% and r_i=10%

Table 6. Degree of closeness with various w_k for case #1 in Table 5

(w_1, w_2, w_3)	Method*	D_1	D_2	D_∞
(0.1,0.1,0.8)	(1)	0.407	0.345	0.341
	(2)	0.397	0.344	0.341
(0.1,0.2,0.7)	(1)	0.406	0.311	0.298
	(2)	0.395	0.310	0.298
(0.1,0.33,0.57)	(1)	0.404	0.280	0.243
	(2)	0.394	0.279	0.243
(0.1,0.4,0.5)	(1)	0.403	0.271	0.213
	(2)	0.393	0.270	0.213
(0.2,0.1,0.7)	(1)	0.389	0.305	0.298
	(2)	0.368	0.303	0.298
(0.2,0.2,0.6)	(1)	0.388	0.273	0.256
	(2)	0.367	0.270	0.256
(0.2,0.33,0.47)	(1)	0.386	0.247	0.200
	(2)	0.365	0.244	0.200

(0.2,0.4,0.4)	(1)	0.385	0.242	0.170
	(2)	0.364	0.239	0.170
(0.33,0.2,0.47)	(1)	0.364	0.231	0.199
	(2)	0.329	0.220	0.199
(0.33,0.33,0.33)	(1)	0.362	0.214	0.142
	(2)	0.327	0.203	0.142
(0.33,0.4,0.26)	(1)	0.361	0.217	0.165
	(2)	0.326	0.206	0.165
(0.33,0.5,0.16)	(1)	0.360	0.233	0.206
	(2)	0.325	0.223	0.206
(0.4,0.2,0.4)	(1)	0.352	0.214	0.170
	(2)	0.310	0.198	0.170
(0.4,0.33,0.26)	(1)	0.350	0.204	0.137
	(2)	0.308	0.187	0.137
(0.4,0.4,0.2)	(1)	0.350	0.210	0.165
	(2)	0.308	0.194	0.165
(0.4,0.5,0.1)	(1)	0.348	0.233	0.206
	(2)	0.306	0.218	0.206

Method*: (1) Fuzzy programming (Liu and Shi, 1994);
(2) Proposed approach

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