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BACKCASTING BITCOIN PRICES: IMPLEMENTATION WITH ARCH & GARCH MODELS

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Abstract

Bitcoin, the first decentralized cryptocurrency, has gained popularity among investors for several reasons. Its potential for high returns makes it attractive to those seeking alternatives to traditional investments. Bitcoin's volatility provides both risk and reward, drawing in speculative investors. Moreover, Bitcoin operates independently of central banks or governments, appealing to those wary of inflation and economic instability. As more businesses and financial institutions adopt Bitcoin as an investment tool and a medium of exchange, its appeal continues to grow. For institutional investors, Bitcoin offers a way to diversify portfolios amid low interest rates and geopolitical uncertainty. However, the volatility in Bitcoin markets tends to be a risk exposure, so developing models to understand Bitcoin fluctuations is crucial to determining more about market behavior. Accurate financial models help predict price movements, manage risk, and identify macroeconomic correlations. Given its complexity, these models are essential for long-term investors to navigate volatility and *optimize their investment strategies. This research employs ARCH and GARCH models to forecast Bitcoin volatility. The outputs indicate that ARIMA is the best fit model that explains Bitcoin's price fluctuations in the selected data period.*

Keywords: Cryptocurrency market, Bitcoin, Modeling, ARCH, GARCH, Forecast

INTRODUCTION

As the pioneering cryptocurrency, Bitcoin has transformed the financial landscape since its launch in 2009. As a decentralized digital currency, it operates on a peer-to-peer network, enabling users to conduct transactions without intermediaries like banks. The volatility of Bitcoin markets, characterized by rapid price fluctuations and significant trading volumes, attracts diverse participants, from individual investors to institutional traders. Understanding the dynamics of Bitcoin markets is crucial for making informed investment decisions. Here, modeling plays a vital role. By employing various analytical techniques, traders and analysts can forecast price movements, assess risk, and identify trading opportunities. Effective modeling helps understand historical trends and simulate potential future scenarios, enhancing strategic decision-making. Numerous factors influence the market, including regulatory changes, technological advancements, and macroeconomic trends. Robust models help navigate the complexities of Bitcoin trading. This introduction sets the stage for a deeper exploration of Bitcoin markets, the methodologies used in their analysis, and the critical role modeling plays in both short-term trading and long-term investment strategies.

LITERATURE REVIEW

Recent literature presents various studies that explore factors influencing cryptocurrency prices and forecasting methodologies. For instance, Naimy and Hayek (2018) analyzed the volatility of the Bitcoin/USD exchange rate by comparing several forecasting models, including GARCH (1,1), EWMA, and EGARCH (1,1). Their findings suggest that Bitcoin's price behavior is unique compared to traditional currencies. Shen et al. (2019) utilized machine learning to forecast Bitcoin's stock price volatility, concluding that neural network analysis outperforms traditional methods like GARCH and simple moving averages. Yıldırım and Bekun (2023) aimed to identify the best model for predicting Bitcoin's return volatility using weekly data from November 2013 to March 2020. After conducting the ADF unit root test for stationarity, they determined ARMA (2,2) as the mean equation model. They explored various variance models, ultimately identifying GARCH (1,1) as the most accurate, providing valuable insights for Bitcoin price analysis. Loureiro (2023) sought to determine the optimal model for examining Bitcoin's price and volatility, finding that the EGARCH (1,1) model was the best fit, underlining the need for models that align with investor risk preferences.

Quan et al. (2023) focused on estimating Bitcoin's volatility using GARCH models and the Box-Jenkins Method, observing that GARCH models capture shock clustering. They also employed GJR-GARCH (1,1), discovering that positive shocks to Bitcoin returns lead to increased return volatility. This indicates that reverse leverage evaluated the ARIMA model's predictive capabilities for Ethereum, particularly during economic shocks like COVID-19. Based on 208 samples of Ethereum data from January 2017 to December 2020, they revealed that ARIMA produced unsatisfactory predictions with notable discrepancies from actual values. Kose et al. (2024) investigated the influences on Bitcoin prices using data including the U.S. dollar index, the Chicago Board Options Exchange's volatility index (VIX), gold prices, oil prices, and Bitcoin price volatility, employing the SVAR model for analysis. Their variance decomposition results highlighted that the VIX's initial impact on Bitcoin prices is limited, with a statistically significant inverse relationship indicated through impulse response functions.

METHODOLOGY

This study investigates future return predictions and directions based on historical Bitcoin prices. It utilizes daily returns computed from Bitcoin's daily closing price covering January 2020 to October 2024. Various financial models have been applied in price prediction studies, particularly nonlinear econometric models like ARCH and GARCH, which account for non-constant variance in economic time series. In this context, the GARCH model is employed to forecast Bitcoin prices. As of October 2024, the study focuses on predicting Bitcoin returns for the last quarter of 2024. To validate the prediction model, known data from July 2024 to October 2024 is excluded, with predictions made and compared to actual returns to ensure accuracy. Following this validation, the GARCH model will be extended to forecast November and December 2024 Bitcoin returns.

Engle's foundational studies on the ARCH model occurred in 1982, 1983, and 1995. Unlike traditional econometric models that assume constant unconditional variance, the ARCH model allows the conditional variance to vary based on prior values of the random variable. In an ARCH (1) structure, the conditional variance (h_t) fluctuates as a function of past squared errors. In contrast, the unconditional variance remains constant under a zero mean assumption, enabling the conditional variance to change over time.

$$Y_t = \Psi_{t-1} \sim N(Y_{t-1}, \beta, h_t) \quad (1)$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \cdots + \alpha_p e_{t-p}^2 \quad (2)$$

$$e_t = Y_t - Y_{t-1}\beta \quad (3)$$

In the GARCH models developed by Tim Bollerslev (1986), the conditional variance (h_t) at period t depends on the square of the previous values of the error terms and the previous conditional variances. Therefore, the variance of the error terms is affected by both the conditional variance values and the past values. Under these conditions:

$$\omega > 0;\ \alpha_i \geq 0;\ \beta_j \geq 0;\ \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \quad (4)$$

With q as the lag length of the error squares and p as the lag length of the autoregressive part, a general GARCH(p,q) process can be described as follows:

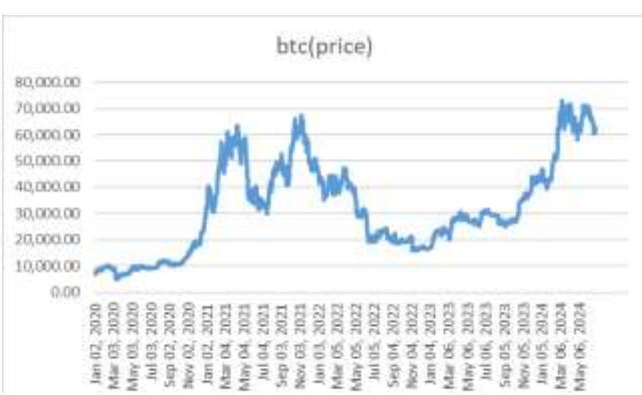
$$h_t = \omega + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \quad (5)$$

As can be easily observed from the equations, the difference between the GARCH model and the ARCH model is that the lags of the conditional variance are also included in the conditional variance equation. Testing the stationarity of the series is crucial for establishing the models in time series analysis and should be conducted before any analysis. Series, which are non-stationary tests, produce false positives (Gujurati, 1999, p. 713). McKinnon (1991) stated that ensuring the stationarity of the series will provide more reliable test results. In this study, Augmented Dickey-Fuller (ADF), Philips-Perron (P.P.), and Kwiatkowski-Philips-Schmidt-Shin (KPSS) unit root tests are used except for the KPSS test. In these tests where unit root tests are performed, the null hypothesis shows the existence of a unit root, while in the KPSS test, the null hypothesis is stationary.

EMPIRICAL RESULTS

This study applied the GARCH model to Bitcoin prices. The main goal is to estimate the returns for the last quarters of 2024 using daily closing prices of Bitcoin from January 2020 to October 2024. The analysis begins with examining the graphs (Figure 1) of the Bitcoin time series and its return series, where the return series is derived by calculating the first-degree difference of the logarithmic series.

Figure 1. Bitcoin Time Series Graphs



Gujarati (2016) notes that financial time series often exhibit non-stationarity due to their high frequency. To address this issue, it is recommended to use the logarithmic return of the financial series to eliminate any trends. The second chart displays the series after it has been de-trended.

Table 1. Unit Root Tests Statistics

Model	ADF	PP	KPSS
intercept	-44.3896	-44.2149	0.2054

*t-statistics for a 5% significance level is -2.86 for ADF and P.P. while 0.4630 for KPSS.

Table 1 shows the probabilities of the “intercept only” models of the unit root tests used in the analysis. According to the table, both ADF, P.P., and KPSS unit root analyses show that the Bitcoin return series is stationary; it does not have a unit root in the first difference. In other words, this result suggests that we should work with return series in the continuation of the analysis.

The ARCH-LM test was used to determine whether there is an ARCH effect in the Bitcoin series after it becomes stationary. Various ARIMA models determine the most suitable model for the series's structure. The appropriate ARIMA model is defined as ARIMA (1,1,1) and shown in the Table 2 below.

Table 2: ARIMA model results

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.0001	0.0001	1.3986	0.1621
AR(1)	-0.6859	0.0965	-7.1006	0.0000
MA(1)	0.6020	0.1035	5.8113	0.0000
SIGMASQ	0.0000	0.0000	117.5877	0.0000

We performed the ARCH-LM test. The results showed an ARCH effect in the model. In this case, BITCOIN returns are suitable for ARCH modeling.

After analyzing the series' stationarity and estimating the average equations for the Bitcoin variable as ARIMA (1,1,1), the estimated volatility models for the Bitcoin return rate are presented in Table 3. ARIMA models cannot adequately determine variance structure asymmetry effects. The GARCH model must be applied to estimate the asymmetric impacts of the shocks on volatility in this situation. The estimation results of the GARCH (1,1) models for the Bitcoin return series appear in Table 3.

Table 3: The Estimation Results of GARCH Model

Mean Equation				
Variable	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
C	0.0002	0.0000	2.9695	0.0030
AR(1)	-0.2296	0.2248	-1.0216	0.3069
MA(1)	0.118	0.2262	0.5239	0.6003
Variance Equation				
C	0.0000	0.0000	11.7446	0.0000
RESID(-1)^2	0.1499	0.0188	7.9373	0.0000
GARCH(-1)	0.5999	0.0267	22.415	0.0000

According to the variance equation of our Bitcoin return model, the reaction and persistence parameters are significant at the 0.05 significance level. The persistence parameter is significantly higher than the reaction parameter. This indicates that a new shock affecting Bitcoin returns will not dissipate quickly; its effects will last for an extended period. In other words, a significant amount of the shock in one period continues to influence the returns in the following periods.

A backward-looking forecast, Backcasting, was made in the next stage of the study. Returns in October 2024 were estimated using Bitcoin return data from January 2020 to September 2024. Weekly forecasts were made for each of the four weeks. To improve accuracy, the best model was determined for every weekly prediction using the ARIMA forecasting method. The correlation coefficient between predicted and actual values was computed weekly to evaluate estimation quality. This made it possible to assess the degree to which forecasts and actual results matched. Table 4 presents a weekly analysis of Bitcoin predicting performance and shows the study's outcomes.

Table 4: Weekly Forecasting for Bitcoin

Descriptive Statistics	**Week 1**		**Week 2**		**Week 3**		**Week 4**	
	actual	***Forecast***	***actual***	***forecast***	***actual***	***forecast***	***actual***	***forecast***
Maximum	0.0019	0.0010	0.005	0.001	0.0045	0.0045	0.0020	0.0004
Minimum	-0.0036	-0.0028	-0.002	-0.001	-0.0021	-0.0003	-0.0021	-0.0021
Std. Dev.	0.0018	0.0014	0.003	0.001	0.0019	0.0016	0.0015	0.0010
Correlation	59.13%		43.39%		85.36%		71.11%	
Model Selected	ARIMA (1,0,0)		ARIMA (4,0,2)		ARIMA (6,0,6)		ARIMA (0,0,4)	

Table 4 contains the essential details of this analysis. The labels "Week 1," "Week 2," "Week 3," and "Week 4" in the table refer to the four weeks of October 2024. The terms "actual" and "forecast" indicate the observed and estimated series. The section titled "Descriptive Statistics" includes "Maximum," "Minimum," and "Standard Deviation (Std. Dev.)," which represent the highest and lowest values of the actual/forecasted series, as well as the related risks. The table also shows the correlation coefficients between the series and the most suitable model used for forecasting.

For Week 1, the analysis determined the most suitable model for forecasting as ARIMA (1,0,0). The minimum and maximum values of both the forecasted and actual data are relatively close, indicating similar levels of risk. The correlation coefficient between the actual and projected values for Week 1 was 59.13%. This result is considered "good," the analysis continued using the same methodology for Week 2.

In analyzing Week 2, the best forecasting model was determined to be ARIMA (4,0,2). Similar to Week 1, the minimum and maximum values of the forecasted and actual values were closely aligned, with the risks being comparable but lower than those of previous weeks. The correlation coefficient for Week 2 was 43.39%, which is also considered "good." As a result, the analysis continued with the same method for Week 3 also.

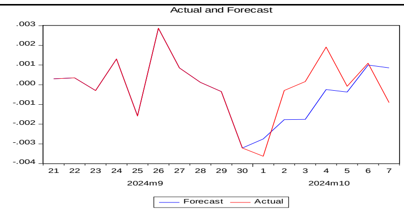
The results for Week 3 indicated that the best forecasting model was ARIMA (6,0,6). This week, the maximum values of the forecasted and actual data were close, while the minimum values showed more significant variation than previous weeks. However, the risks remained similar. The correlation coefficient for Week 3 was notably high at 85.36%, indicating a strong positive relationship. This result was considered "better," leading to further analysis for Week 4.

Lastly, for Week 4 (which contains 5 days compared to the 7 days of the previous weeks), the most suitable model was ARIMA(0,0,4). The minimum values of both forecasted and actual figures were close, although the maximum values were relatively distant. The risks were similar. The correlation coefficient for Week 4 is 71.11%, reflecting a positive relationship and considered "better."

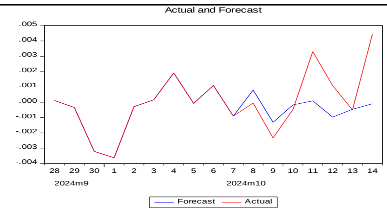
As shown, backward-looking ARIMA forecasting provided reasonable results for short-term predictions of a highly volatile Bitcoin series. To visualize the findings presented in Table 4, Figure 2 is created, showing the weekly forecasting results graphically.

Figure 2: Weekly Forecasting Graphs

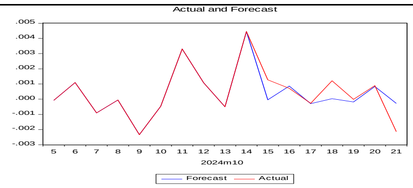
Week 1



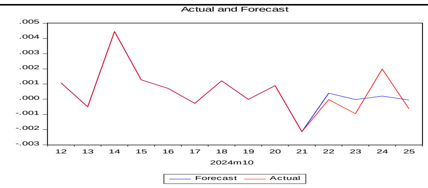
Week 2



Week 3



Week 4



CONCLUSION

Bitcoin has increased in popularity and attracted investors' attention since its launch in 2009. It is known that the market grew more strongly, especially after the pandemic. With the ongoing low interest rate environment in global markets during the pandemic, individual and institutional investors have turned their investment interests into more risky assets. These conditions have brought about a huge investor demand for crypto markets and caused price fluctuations. This study focuses on modeling the price volatility of Bitcoin between January 2020 to September 2024. The daily return data is used, and the main aim is to test whether ARCH and GARCH models can be used for a forecast. Initially, the stationarity of the time series was tested. Then, the ARCH-LM test was used to determine whether there is an ARCH effect in the Bitcoin series. The results indicated that an ARCH effect exists in the model. Since ARIMA models cannot adequately determine variance structure asymmetry effects, the GARCH model must be applied to estimate the asymmetric impacts of the shocks on volatility. Finally, backcasting is implemented. The daily returns were calculated weekly, and actual vs. forecast returns were compared.

REFERENCES

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307-327.

Engel C, Frankel JA, Froot KA, Rodrigues AP. (1995). Tests of conditional mean-variance efficiency of the U.S. stock market. Journal of Empirical Finance 2: March 3–18.

Engle, Robert F. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation." Econometrica, July 1982, 50(4), pp. 987-10.

Engle, Robert F. "Estimates of the Variance of U.S. Inflation Based Upon the Arch Model." Journal of Money, Credit, and Banking, August 1983, 15(3), pp. 286-3.

Gujurati, D. N. (1999). Basic Econometrics Mc Graw Hill. İstanbul: Literatür Yayıncılık.

Gunawan, D., & Febrianti, I. (2023). Ethereum Value Forecasting Model using Autoregressive Integrated Moving Average (ARIMA). International Journal of Advances in Social Sciences and Humanities, 2(1), 29-35.

Köse, N., Yildirim, H., Ünal, E., & Lin, B. (2024). The Bitcoin price and Bitcoin price uncertainty: Evidence of Bitcoin price volatility. Journal of Futures Markets.

Loureiro, C. V. (2023). Forecasting Bitcoin returns volatility using GARCH methods.

McKinnon, R. I. (1991). Financial control in the transition from classical socialism to a market economy. Journal of Economic Perspectives, 5(4), 107-122.

Naimy, V. Y., & Hayek, M. R. (2018). Modeling and predicting the Bitcoin volatility using GARCH models. International Journal of Mathematical Modelling and Numerical Optimisation, 8(3), 197-215.

Quan, Y. X., Yang, T. X., Fei, C. Y., Cheong, C. W., & Min, L. (2023). Asymmetrıc Volatılıty And Rısk Analysıs Of The Bıtcoın Cryptocurrency Market. Journal of Quality Measurement and Analysis JQMA, 19(2), 73-79.

Shen, Z., Wan, Q., & Leatham, D. J. (2019). Bitcoin Return Volatility Forecasting: A Comparative Study of GARCH Model and Machine Learning Model.

Yıldırım, H., & Bekun, F. V. (2023). Predicting volatility of bitcoin returns with ARCH, GARCH, and EGARCH models. Future Business Journal, 9(1), 75.