COMPETITION BETWEEN A PUBLIC AND A PRIVATE HOSPITALS WITH TAXATION

Wei-Wei Lee ✉

Associate Professor, Department of Economics, National Dong Hwa University, Taiwan

wwlee@gms.ndhu.edu.tw

Ping-Ru Huang

Graduate student, Department of Economics, National Dong Hwa University, Taiwan

610242015@ems.ndhu.edu.tw

Abstract

This study examines the impact of taxation on the quality and pricing strategies of public and private hospitals and its subsequent effects on social welfare. The results indicate that the quality of healthcare in public hospital improves with a small tax but declines with medium or large tax rates. A private hospital, on the other hand, experiences a decrease in healthcare quality with any tax imposition. Both types of hospitals adjust their pricing similarly, lowering prices with small to medium taxes and increasing them with a large tax, affecting their profits accordingly. Social welfare tends to improve with a large tax rate within a limited range; otherwise, it declines. A small tax leads to decreased welfare due to intensified competition, while a large tax reduces consumer surplus and healthcare quality, impacting social welfare negatively. A tax does not simultaneously raise both healthcare quality and social welfare. Therefore, the government should be cautious in designing tax policies.

Keywords: taxation, public hospital, private hospital, price competition, quality competition

INTRODUCTION

With the rapid advancement of medical technologies, the average human life expectancy has steadily increased, resulting in a significant rise in medical expenditures for the nation. This underscores the growing importance placed on healthcare by individuals, ranging from proactive health maintenance to post-illness care and a myriad of health assessments. However, it is important to note that each patient's experience with the same doctors within the same institution can vary widely. Consequently, to cater to the diverse needs of consumers, medical institutions strive to offer a range of products differing in quality.

The healthcare market differs from general markets as it may require government interventions to balance hospital profits and healthcare quality, a target which could be jeopardized under intense competition. It is well noted that public and private hospitals hold different objectives: the former may pursue both profits and patient well-beings, while the latter focuses on its own profits. This prompts intriguing inquiries into how the competition between the two hospital types affects the quality of healthcare and social welfare. Furthermore, discussing how government interventions can help improving healthcare quality is another aspect worthy of attention. To address these compelling issues, this study employs a vertical differentiation model to investigate the competition between the two types of hospitals, incorporating the effects of taxation.

In the relevant literature, Hotelling (1929) investigates price competition in a duopoly market characterized by horizontal differentiation stemming from location choices. Many subsequent articles have drawn upon Hotelling's model as a foundational framework to expand upon and delve deeper into these concepts. Cremer and Thisse (1991) utilize the vertical differentiation model to examine the competition between public and private firms. Their findings indicate that the horizontal differentiation model, which focuses on location choices, can be regarded as a specific case within the framework of vertical differentiation.

In a different context, Brekke et al. (2006) investigate the effects of price control policies in a duopoly market, revealing that social welfare can be enhanced when the government implements price restrictions before firm competition. Furthermore, Sanjo (2007) uses a location model to examine the competition between public and private firms, taking into account factors such as location, quality, and pricing. Cremer and Thisse (1994) examine the impact of commodity tax on the competition in a vertically differentiated market. They discover that both high-quality and low-quality firms reduce quality with a tax. The social welfare can be enhanced by a small amount of tax.

Concerning potential policies in the medical market, Brekke et al. (2012) investigate the effects of profit constraints with price controls. They find that under price controls, profit

constraints could enhance quality of services provided by the firms and could raise overall social welfare, depending the degree of altruism. In a related context, Herr (2011) explore the effects of various objectives on quality, profit, and overall welfare in a duopoly medical market with price restriction policies. The results suggest that privatizing the public hospital could lead to an increase in social welfare.

Research in the medical industry has focused on the implementation of price restriction policies. Nevertheless, taxation emerges as another feasible policy option. Hence, this study refers to Cremer and Thisse (1994) to investigate the effects of taxation on the quality of healthcare services provided by public and private hospitals.

This study finds that a small tax imposition prompts the public hospital to enhance the healthcare quality to cater to a broader consumer base. However, when facing a medium to large tax, the public hospital tends to lower quality in light of cost considerations. Conversely, private hospitals tend to diminish quality whenever subjected to taxation.

Both public and private hospitals adjust their pricing strategies in a similar way. With a small to medium tax, both types of hospitals decrease prices to stimulate demand. In the face of a large tax, both hospitals transfer the tax burden to consumers through price increases, consequently raising hospital profits. This pattern extends to profits, where a small to moderate tax leads to diminished profits due to lowered prices, while a large tax results in increased profits owing to elevated prices.

As for social welfare, it tends to improve with a large tax rate, albeit within a very limited range of tax rates. Otherwise, social welfare declines. When a small tax is imposed, the decrease in welfare can be attributed to diminished profits arising from intense competition. Conversely, if a substantial tax is imposed, the decline in welfare results from the reduction of consumer surplus, which is driven by a decrease in the quality of healthcare provided by both hospitals.

This paper is organized as follows: Section 1 is the introduction; Section 2 explains the basic model; Section 3 derives the equilibrium with a tax and provides the comparative static analysis; Section 4 concludes the paper.

THE MODEL

Basic settings

In the healthcare market, there are two hospitals: Hospital 0, a public institution characterized as a low-quality firm, and Hospital 1, a private facility positioned as a high-quality firm. The production costs for both hospitals can be expressed as $C_i = \frac{1}{2}q_i^2, i = [0,1]$. q_i represents the quality level of Hospital i, with $q_i > 0$.

Consumer are evenly distributed along the range $[\bar{\theta} - 1, \bar{\theta}]$, with $\bar{\theta} > 1$, based on their willingness to pay for quality, θ. The utility function of consumer θ can be written as:

$$U_\theta = \theta q_i - p_i$$

We can find the marginal consumer who is indifferent between receiving treatment from Hospital 0 and Hospital 1 as:

$$\theta^* = \frac{p_1 - p_0}{q_1 - q_0} \tag{1}$$

The demand functions for the public and the private hospitals are:

$$D_0 = \theta^* - \bar{\theta} + 1 \tag{2}$$

$$D_1 = \bar{\theta} - \theta^* \tag{3}$$

The consumer surplus from obtaining healthcare services at Hospital 0 and Hospital 1 is given as follows:

$$CS_0 = \int_{\bar{\theta}-1}^{\theta^*} (\theta q_0 - p_0) d\theta \tag{4}$$

$$CS_1 = \int_{\theta^*}^{\bar{\theta}} (\theta q_1 - p_1) d\theta \tag{5}$$

The profit function of two hospitals are:

$$\pi_i = \left(p_i - \frac{1}{2} q_i^2 \right) D_i, \quad i = 0,1 \tag{6}$$

Social welfare is defined as the sum of consumer surplus and the profits of the two hospitals:

$$W = \int_{\bar{\theta}-1}^{\theta^*} (\theta q_0 - p_0) d\theta + \int_{\theta^*}^{\bar{\theta}} (\theta q_1 - p_1) d\theta + \pi_0 + \pi_1 \tag{7}$$

The objective of the private hospital (Hospital 1) is profit maximization, while the public hospital (Hospital 0) aims to maximize social welfare.

Market equilibrium

The two hospitals engage in a two-stage game. In the first stage, they determine the quality simultaneously, and in the second stage, they decide the price at the same time. Backward induction is applied to find the equilibrium.

Price competition

In the price competition stage, the first order conditions are as follows:

$$\frac{\partial W}{\partial p_0} = \left(\frac{p_1 - p_0}{q_1 - q_0} \right) - \frac{1}{2}(q_0 + q_1) = 0 \tag{8}$$

$$\frac{\partial \pi_1}{\partial p_1} = \overline{\theta} - \frac{2p_1 - p_0}{q_1 - q_0} + \frac{1}{2}q_1^2 \frac{1}{q_1 - q_0} = 0 \tag{9}$$

Through (8) and (9), the prices of the two hospitals can be obtained as:

$$p_0 = (q_1 - q_0)\overline{\theta} - \frac{1}{2}q_1^2 + q_0^2 \tag{10}$$

$$p_1 = (q_1 - q_0)\overline{\theta} + \frac{1}{2}q_0^2 \tag{11}$$

Quality competition

Using (10) and (11), we can derive the objective functions for the public hospital and the private hospital as follows:

$$W = -\frac{1}{8}q_0^3 + \frac{1}{2}\overline{\theta}q_0^2 - \frac{1}{2}q_0^2 - \frac{1}{8}q_0^2 q_1 - \frac{1}{2}q_0 + \frac{1}{8}q_1^2 q_0 - \frac{1}{2}q_0\overline{\theta}^2 + \overline{\theta}q_0 + \frac{1}{8}q_1^3 + \frac{1}{2}\overline{\theta}^2 q_1$$
$$- \frac{1}{2}\overline{\theta}q_1^2 \tag{12}$$

$$\pi_1 = \left[\overline{\theta}(q_1 - q_0) - \frac{1}{2}(q_1 - q_0)(q_1 + q_0)\right]\left[\overline{\theta} - \frac{1}{2}(q_1 + q_0)\right] \tag{13}$$

The first-order conditions are:

$$\frac{\partial W}{\partial q_0} = -\frac{1}{8}(3q_0 - q_1 + 2 - 2\overline{\theta})(q_1 + q_0 + 2 - 2\overline{\theta}) = 0 \tag{14}$$

$$\frac{\partial \pi_1}{\partial q_1} = \left[\overline{\theta} - q_1 - \frac{1}{2}(q_1 - q_0)\right]\left[\overline{\theta} - \frac{1}{2}(q_1 + q_0)\right] \tag{15}$$

By (14) and (15), the equilibrium levels of quality of both hospitals can be solved as:

$$q_0 = \overline{\theta} - \frac{3}{4} \tag{16}$$

$$q_1 = \overline{\theta} - \frac{1}{4} \tag{17}$$

Therefore, the prices for the two hospitals are:

$$p_0 = \frac{1}{2}\overline{\theta}^2 - \frac{3}{4}\overline{\theta} + \frac{17}{32} \tag{18}$$

$$p_1 = \frac{1}{2}\overline{\theta}^2 - \frac{1}{4}\overline{\theta} + \frac{9}{32} \tag{19}$$

Based on (16) ~ (19), we are able to obtain the consumer surplus, profits of the two hospitals, and social welfare are as follows.[1]

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[1] To make the model plausible, it is necessary to satisfy the condition: $\overline{\theta} > \frac{3}{4} + \frac{1}{2}\sqrt{2}$.

The Consumer surplus of receiving treatment at hospital 0 is:

$$CS_0 = \frac{1}{4}\overline{\theta}^2 - \frac{3}{8}\overline{\theta} + \frac{1}{64}. \tag{20}$$

Similarly, the consumer surplus of receiving treatment at and hospital 1 is:

$$CS_1 = \frac{1}{4}\overline{\theta}^2 - \frac{1}{8}\overline{\theta} - \frac{7}{64} \tag{21}$$

Profits of the two hospitals are:

$$\pi_0 = \frac{1}{8} \tag{22}$$

$$\pi_1 = \frac{1}{8} \tag{23}$$

Social welfare is:

$$W = \frac{1}{2}\overline{\theta}^2 - \frac{1}{2}\overline{\theta} + \frac{5}{32} \tag{24}$$

EQUILIBRIUM WITH TAX AND COMPARATIVE STATIC ANALYSIS

Equilibrium with Tax

In this section, we explore the effects of taxation on price, quality, social welfare, and the profits of both public and private hospitals. Variables with subscripts "t" denote the case under taxation.

The profit functions of the two hospitals with tax are:

$$\pi_{it} = \left(p_{it}(1-t) - \frac{1}{2}q_{it}^2\right)D_{it}, i = 0,1 \tag{25}$$

Social welfare is defined as:

$$W_t = \int_{\overline{\theta}-1}^{\theta^*} (\theta q_{0t} - p_{0t})d\theta + \int_{\theta^*}^{\overline{\theta}} (\theta q_{1t} - p_{1t})d\theta + \pi_{0t} + \pi_{1t} \tag{26}$$

Price Competition

The first order conditions at this stage are:

$$\frac{\partial W_t}{\partial p_{0t}} = t\overline{\theta} - \frac{1}{2}(q_{1t} + q_{0t}) - t - \frac{(2t-1)(p_{1t} - p_{0t})}{q_{1t} - q_{0t}} = 0 \tag{27}$$

$$\frac{\partial \pi_{1t}}{\partial p_{1t}} = \overline{\theta} - t\overline{\theta} - \frac{(t-1)(2p_{1t} - p_{0t})}{q_{1t} - q_{0t}} + \frac{1}{2}q_{1t}^2\frac{1}{q_{1t} - q_{0t}} = 0 \tag{28}$$

Thus, the equilibrium prices with tax at this stage are:

$$p_{0t} = \frac{2(\overline{\theta} + 2t^2)(q_{0t} - q_{1t}) + 2t(\overline{\theta} + 2)(q_{1t} - q_{0t}) + 2q_{0t}^2(t-1) + q_{1t}^2}{-2(-1+2t)(-1+t)} \tag{29}$$

$$p_{1t} = \frac{2t(2t + t\overline{\theta} - 1)(q_{0t} - q_{1t}) + 2\overline{\theta}(2t - 1)(q_{1t} - q_{0t}) + q_0^2(t - 1) + tq_{1t}^2}{-2(-1 + 2t)(-1 + t)} \tag{30}$$

Quality Competition

Substituting (29) and (30) into the objective functions, we can have:

$$W_t = \frac{1}{8(-1 + 2t)(-1 + t)}(-4t^2\overline{\theta}q_{1t}^2 - 8t\overline{\theta}q_{1t} - 16t\overline{\theta}q_{0t} + 8\overline{\theta}q_{0t} - q_{0t}^3 + q_{1t}^3$$

$$-4\overline{\theta}q_{1t}^2 + 4\overline{\theta}q_{0t}^2 + 12tq_{0t} + 4tq_{1t}^2 + 4tq_{0t}^2 - q_{0t}^2 q_{1t} + q_{1t}^2 q_{0t} + 8t\overline{\theta}q_{1t}^2 + tq_{0t}^2 q_{1t}$$

$$-8t\overline{\theta}q_{0t}^2 + 4t^2\overline{\theta}q_{0t}^2 - 4q_{0t} + 4q_{0t}\overline{\theta}^2 t^3 - 4q_{0t}^2 - 4q_{1t}\overline{\theta}^2 t^3 - tq_{1t}^3 + 8q_{0t}\overline{\theta}t^2$$

$$+8q_{1t}\overline{\theta}t^2 - 20t^2 q_{0t} + 12t^2 q_{1t} - 12t^3 q_{1t} + 12t^3 q_{0t} - tq_{0t}q_{1t}^2 - 12q_{0t}\overline{\theta}^2 t^2$$

$$+tq_{0t}^3 - 12q_{1t}\overline{\theta}^2 t + 12q_{1t}\overline{\theta}^2 t^2 + 4q_{1t}\overline{\theta}^2 + 12q_{0t}\overline{\theta}^2 t - 4q_{0t}\overline{\theta}^2) \tag{31}$$

$$\pi_{1t} = \frac{(-1 + t)(q_{0t} - q_{1t})(q_{0t} + q_{1t} + 2t + 2t\overline{\theta} - 2\overline{\theta})^2}{4(-1 + 2t)^2} \tag{32}$$

Differentiating (31) and (32) with respect to the regarding quality gives the two first order conditions at the stage of quality competition. Solving these first-order conditions provides the equilibrium qualities as:

$$q_{0t} = \frac{1}{8}t - t\overline{\theta} + \overline{\theta} + \frac{3}{8}\sqrt{-23t^2 + 14t + 1} - \frac{9}{8} \tag{33}$$

$$q_{1t} = -\frac{5}{8}t - t\overline{\theta} + \overline{\theta} + \frac{1}{8}\sqrt{-23t^2 + 14t + 1} - \frac{3}{8} \tag{34}$$

Therefore, the equilibrium prices of the two hospitals under taxation are:

$$p_{0t} = \frac{1}{64(2t - 1)(-1 + t)}(-128\overline{\theta}^2 t + 32\overline{\theta}^2 + 160\overline{\theta}^2 t^2 - 64\overline{\theta}^2 t^3 + 224\overline{\theta}t - 72\overline{\theta}$$

$$-168\overline{\theta}t^2 + 24\overline{\theta}\sqrt{-23t^2 + 14t + 1} - 72\overline{\theta}t\sqrt{-23t^2 + 14t + 1} + 16\overline{\theta}t^3$$

$$+48\overline{\theta}t^2\sqrt{-23t^2 + 14t + 1} + 97t\sqrt{-23t^2 + 14t + 1} + 110t^3 - 123t^2$$

$$-38t^2\sqrt{-23t^2 + 14t + 1} - 51\sqrt{-23t^2 + 14t + 1} - 100t + 851) \tag{35}$$

$$p_{1t} = \frac{1}{64(2t - 1)(-1 + t)}(-128\overline{\theta}^2 t + 32\overline{\theta}^2 + 160\overline{\theta}^2 t^2 - 64\overline{\theta}^2 t^3 + 32\overline{\theta}t - 24\overline{\theta}$$

$$-24\overline{\theta}t\sqrt{-23t^2 + 14t + 1} + 16\overline{\theta}t^2\sqrt{-23t^2 + 14t + 1} + 8\overline{\theta}t\sqrt{-23t^2 + 14t + 1}$$

$$-80\overline{\theta}t^3 + 72\overline{\theta}t^2 + 49t\sqrt{-23t^2 + 14t + 1} - 14t^2\sqrt{-23t^2 + 14t + 1} - 83t^2$$

$$+54t^3 - 44t - 27\sqrt{-23t^2 + 14t + 1} + 45) \tag{36}$$

Subsequently, we can have profits of the two hospitals[2]:

$$\pi_{0t} = \frac{1}{16(3t - 3 + \sqrt{-23t^2 + 14t + 1})(2t - 1)^2}(19t^2 - 16t + t\sqrt{-23t^2 + 14t + 1}$$

$$+\sqrt{-23t^2 + 14t + 1} + 1)(12t^3 - 11t^2 + 4t^2\sqrt{-23t^2 + 14t + 1} + 8t$$

$$-5t\sqrt{-23t^2 + 14t + 1} + 3\sqrt{-23t^2 + 14t + 1} - 5) \tag{37}$$

$$\pi_{1t} = \frac{3\sqrt{-23t^2 + 14t + 1} - 7t^2 - 2t + 5}{16(3t - 3 + \sqrt{-23t^2 + 14t + 1})(2t - 1)^2} \tag{38}$$

The social welfare under a tax is:

$$W_t = -\frac{1}{128(2t - 1)(-1 + t)}(124t^4 - 128t^3\overline{\theta}^2 - 252t^3 + 43t^3\sqrt{-23t^2 + 14t + 1}$$

$$+128t^3\overline{\theta} - 101t^2\sqrt{-23t^2 + 14t + 1} + 320t^2\overline{\theta}^2 + 318t^2 - 320t^2\overline{\theta} - 256t\overline{\theta}^2$$

$$+256t\overline{\theta} - 152t + 39t\sqrt{-23t^2 + 14t + 1} + 64\overline{\theta}^2 - 64\overline{\theta} + 3\sqrt{-23t^2 + 14t + 1}$$

$$+18) \tag{39}$$

Based on the equilibrium levels of the above variables, we can perform a comparative static analysis to observe the effects of taxation on these variables.

Comparative Statics

The section discusses how the impact of taxation on healthcare qualities, prices, and profits of both hospital, and on social welfare.[3] The tax is categorized into three sections for discussion: a small amount of tax for $0 < t < 0.15$, a medium amount of tax for $0.15 < t < 0.5$, and a large amount of tax for $0.5 < t < 0.67$.

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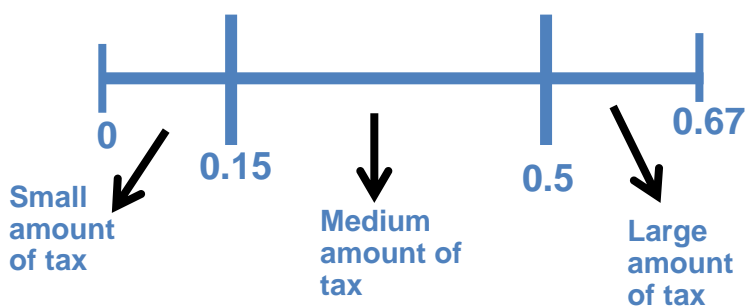
[2] In the following analysis, $k = \sqrt{-23t^2 + 14t + 1}$. $0 < t < 0.67, t \neq 0.5$. The hospitals will leave the market if $t > 0.67$.
[3] The results of the comparative static analysis are based on simulations.

Lemma 1. For $0 < t < 0.15$, the quality of the public hospital increases as t increases.   For 0.15< t < 0.67 (t≠0.5), the quality of the public hospital decreases as t increases.

Proof:

$$\frac{\partial q_{0t}}{\partial t} = \frac{1}{8} - \overline{\theta} + \frac{3}{16}\frac{-46t+14}{k} \quad \begin{cases} > 0, & if \ 0 < t < 0.15 \\ < 0, & if \ 0.15 < t < 0.67 \ (t \neq 0.5) \end{cases}$$

When t is less than 0.15, imposing a small amount of tax prompts the public hospital to enhance quality to meet the needs of a larger consumer base. However, when t exceeds 0.15, the taxation costs lead the public hospital to reduce quality, aiming to cut costs and enlarge the quality gap, subsequently reducing market competition.

Lemma 2. Taxation reduces the quality of the private hospital.

Proof:

$$\frac{\partial q_{1t}}{\partial t} = -\frac{5}{8} - \overline{\theta} + \frac{1}{16}\frac{-46t+14}{k} < 0$$

In response to a tax, the private hospital would lower its quality to reduce costs, consequently raising profits.

By Lemma 1 and Lemma 2, in the cases of a medium and a large amount of tax, the conclusions on the qualities provided by both hospitals here are in line with those in Cremer and Thisse (1994) wherein both firms reduce qualities.

Lemma 3. As t increases, the price of the public hospital decreases when 0 < t < 0.5, while it rises when 0.5 < t < 0.67.

Proof:

$$\frac{\partial p_{0t}}{\partial t} = \frac{1}{-64k(2t-1)^2(-1+t)^2}(-192\overline{\theta}^2 tk + 416\overline{\theta}^2 t^2 k - 384\overline{\theta}^2 t^3 k + 128\overline{\theta}^2 t^4 k$$

$$+32\overline{\theta}^2 k + 48\overline{\theta}tk - 32\overline{\theta}t^4 k + 96\overline{\theta}t^3 k - 104\overline{\theta}t^2 k - 8\overline{\theta}k - 168\overline{\theta} + 660t^3 k$$

$$-889t^2 k - 220t^4 k + 1560\overline{\theta}t - 5496\overline{\theta}t^2 + 9192\overline{\theta}t^3 - 7296\overline{\theta}t^4 + 2208\overline{\theta}t^5$$

$$+586tk + 5767t^2 - 8005t^3 + 5776t^4 - 1748t^5 - 2267t - 155k + 413)$$

$$\begin{cases} < 0, & if \ 0 < t < 0.5 \\ > 0, & if \ 0.5 < t < 0.67 \end{cases} \quad\quad\quad\quad (40)$$

When the tax is below 0.5, the public hospital would lower the price to stimulate demand. However, when the tax ranges between 0.5 and 0.67, the public hospital would increase the price to partially pass the tax burden to consumers. In the case of large amount of tax, the profit derived from raising prices will be higher than that obtained from lowering prices.

Lemma 4. For $0 < t < 0.5$, *the price of the private hospital falls as t increases. For* $0.5 < t < 0.67$, *the prices private of the hospital rises as t increases.*

Proof:

$$\frac{\partial p_{1t}}{\partial t} = \frac{1}{64(2-1)(-1+t)}(32\bar{\bar{\theta}}^2 t + 32\bar{\bar{\theta}}^2 - 128\bar{\bar{\theta}}^2 t + 160\bar{\bar{\theta}}^2 t^2 - 64\bar{\bar{\theta}}^2 t^3 - 24\bar{\bar{\theta}}$$

$$+32\bar{\theta}t + 8\bar{\theta}k + 72\bar{\theta}t^2 - 80\bar{\theta}t^3 + 16\bar{\theta}t^2 k - 24\bar{\theta}tk + -83t^2 - 14t^2 k - 44t$$

$$+54t^3 + 49tk + 45 \begin{cases} < 0, & if\ 0 < t < 0.5 \\ > 0, & if\ 0.5 < t < 0.67 \end{cases} \tag{41}$$

Similar to the public hospital, when the tax is less than 0.5, the private hospital would reduce the prices to increase demand. However, in the tax range of 0.5 to 0.67, the private hospital would raise prices to increase its profits by partially shifting the tax burden to the consumers.

In Cremer and Thisse (1994), in response to reduced qualities, firms choose to lower prices in the presence of taxation. In contrast to the above findings, our study shows that in the cases of a small and a medium amount of tax, both hospitals tend to decrease prices. The public hospital, with a focus on enhancing social welfare, reduces prices to accommodate more patients, provided it remains financially feasible. Meanwhile, following Lemma 2, which indicates a decline in quality with taxation for the private hospital, it decreases prices in accordance with the reduced quality. Failure to do so would risk losing consumers to the public counterpart.

Lemma 5. As t increases, the profit of the public hospital decreases when 0 < t < 0.5, while it rises when 0.5 < t < 0.67.

Proof:

$$\frac{\partial \pi_{0t}}{\partial t} = -\frac{1}{2(3t-3+k)^2(2t-1)^3 k}(-7t^2 + 3tk - 2t - 3k + 5)(23t^5 - 160t^4$$

$$+45t^4 k + 240t^3 - 99t^3 k - 151t^2 + 81t^2 k - 36tk + 59t + 9k - 15)$$

$$\begin{cases} < 0, & if\ 0 < t < 0.5 \\ > 0, & if\ 0.5 < t < 0.67 \end{cases} \tag{42}$$

When the tax rate ranges between 0.5 to 0.67, in accordance with Lemmas 1 and 3, the public hospital raises prices and reduces quality (resulting in lower costs), thereby achieving a higher level of profit. However, if the tax rate is below 0.5, the combination of reduced prices and improved quality leads to a decrease in profit for the public hospital.

Lemma 6. The profit of the private hospital decreases when 0<t<0.5, while it increases when 0.5<t<0.67.

Proof:

$$\frac{\partial \pi_{1t}}{\partial t} = -\frac{1}{4(3t-3+k)^2(2t-1)^3 k}(9200t^7 - 32614t^6 + 1616t^6 k + 46923t^5$$

$$-3570t^5 k - 35345t^4 + 2775t^4 k - 908t^3 k + 120t^2 k - 4998t^2 + 138tk$$

$$761t - 103k + 89 \begin{cases} < 0, & if\ 0 < t < 0.5 \\ > 0, & if\ 0.5 < t < 0.67 \end{cases} \tag{43}$$

When the tax is below 0.5, in accordance with Lemma 2 and 4, the private hospital reduces prices and quality. While costs decrease due to the lower quality, the magnitude of the price reduction exceeds the decrease in cost, resulting in a decline in the profit. Conversely, when the tax is between 0.5 and 0.67, similar to the public hospital, the private hospital raises prices and lowers quality, leading to an increase in the profit.

Lemma 7. Welfare rises in t when $0.5 < t < 0.54$.

Proof:

$$\frac{\partial W}{\partial t} = -\frac{1}{16(2t-1)^2(-1+t)^2(3t-3+k)^2 k}(-4416\overline{\theta}^2 t^7 + 20352\overline{\theta}^2 t^6$$

$$-448\overline{\theta}^2 t^6 k - 38160\overline{\theta}^2 t^5 + 1216\overline{\theta}^2 t^5 k + 37008\overline{\theta}^2 t^4 - 752\overline{\theta}^2 t^4 k - 704\overline{\theta}^2 t^3$$

$$-19296\overline{\theta}^2 t^3 + k4896\overline{\theta}^2 t^2 + 1120\overline{\theta}^2 t^2 k - 336\overline{\theta}^2 t - 48\overline{\theta}^2 + 80\overline{\theta}^2 k$$

$$-512\overline{\theta}^2 k + 4416\overline{\theta}t^7 - 20352\overline{\theta}t^6 - 1216\overline{\theta}t^5 k + 38160\overline{\theta}t^5 + 752\overline{\theta}t^4 k$$

$$-37008\overline{\theta}t^4 + 704\overline{\theta}t^3 k + 19296\overline{\theta}t^3 - 1120\overline{\theta}t^2 k - 4896\overline{\theta}t^2 + 512\overline{\theta}tk$$

$$+336\overline{\theta}t + 448\overline{\theta}k - 80\overline{\theta}k + 48\overline{\theta} + 5336t^8 + 2248t^7 k - 28762t^7 - 9910t^6 k$$

$$+66842t^6 + 18248t^5 k - 93165t^5 + 82793t^4 - 17775t^4 k + 8796t^3 k$$

$$-44304t^3 - 1336t^2 k + 12280t^2 - 48tk - 1017t + 157k - 131k$$

$$\begin{cases} < 0, & if\ 0 < t < 0.5 \\ > 0, & if\ 0.5 < t < 0.54 \\ < 0, & if\ 0.5 < t < 0.67 \end{cases} \tag{44}$$

When the tax lies between 0.5 and 0.54, prices increases and quality decreases lead to a significant rise in profits for both hospitals and, consequently, an increase in social welfare. Conversely, when the tax is less than 0.5, the reduction in prices results in a drop in profits for both hospitals, leading to a decrease in social welfare. Additionally, when the tax exceeds 0.54, the substantial increase in prices reduces consumer surplus, contributing to a decline in social welfare. Thus, welfare improves only within a narrow tax range.

The imposition of a tax, relatively low or relatively high, is found to be welfare-reducing in this study. A small tax narrows the quality gap, intensifying competition so as to reducing hospital profits and welfare. With a tax falls in the scope of large tax (within a very limited range

on the left edge of this spectrum), the quality gap expands, leading to increased hospital profits and a potential rise in social welfare. The above result on welfare is in line with Cremer and Thisse (1994). However, in contrast to this scenario, both hospitals reduce quality under a substantial tax, resulting in a significant increase in prices and a substantial loss in consumer surplus, leading to a lower level of welfare. Thus, social welfare improves with a large tax within a very limited range of tax rates in this study.

In addition, there exists a certain range of tax rate that can improve both hospital profits and social welfare. However, imposing a tax does not simultaneously raise both healthcare quality and welfare. Therefore, the government should exercise caution and conduct thorough assessments when formulating tax policies.

CONCLUSIONS

This study explores the impact of taxation on the healthcare qualities and prices of both low-quality public hospital and high-quality private hospital in a fully covered market. The resulting changes in hospital profits and social welfare are also examined.

The results indicate that the low-quality public hospital raises qualities while the high-quality counterpart lowers it if a small amount of tax is imposed, and both hospitals reduce quality provided with a medium to large tax rate. As for prices and profits, a small to medium tax leads to a decrease, while a large tax result in an increase in both.

A tax is welfare-reducing unless the tax rate falls in the very limited range on the left edge of the scope of large tax. Moreover, a tax does not simultaneously raise both healthcare quality and social welfare. Therefore, the government should be cautious in designing tax policies.

A tax is welfare-reducing unless the tax rate falls in the very limited range on the left edge of the scope of large tax. Moreover, a tax does not simultaneously raise both healthcare quality and social welfare. In the case of a mixed duopoly where the public hospital already focuses on social welfare, implementing a tax may not be an effective policy instrument. Therefore, the government should be cautious in designing tax policies to ensure positive impacts on social welfare. Alternatively, the government could consider other policies such as price controls, which may facilitate quality competition and thereby enhance welfare.

This study did not discuss the possible impact of the location of medical institutions, but in the literature of quality competition, location selection is also one of the directions that can be paid attention to. As hospitals may use location to reduce the degree of competition, the impact of taxes on medical qualities, profits and welfare may vary. Therefore, adding the selection of location can facilitate the discussions in more dimensions.

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