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CHAOS, THE FINANCIAL MARKETS, AND SYMMETRY

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Abstract

Efficient Market Hypothesis (EMH) states that share prices reflect all information, and that therefore, nobody can beat the market [1]. But we know that investors such as Warren Buffett and Peter Lynch can and do beat the market. Moreover, phenomena exist in financial markets that EMH does not explain, such as cycles, trends, one-day 500-point moves, and so on. Recent evidence of “chaos” in financial markets exists [2, p.121]; the author tests the inefficient market (IEM) hypothesis [3] by applying mathematical knot theory [4,5] in its relation to chaos theory. This article also addresses some aspects of the knot in the financial chart, such as the wild knot and symmetry of the knot. A knot may be seen as a chaotic attractor that can provide insights as to price action. Knots are created in evolution of price in time; entropy [6] of the chaotic attractors increases with respect to time. Disorder of the chaotic attractors is a result of entropy, resulting in the evolution of price in time appearing like a traveling wave in a network or lattice of attractors. The attractors can indicate support and resistance levels. Soliton waves [7] also are found in FM charts. These concepts are applied to chart examples to shed light on trends, cycles, extreme volatility (FM), and security price prediction.

Keywords: Symmetry, Knots, Inefficient Market, Entropy, Chaotic Attractors, Soliton Waves, Trends, Cycles

INTRODUCTION

Chaos

Physicist Henri Poincare in the early twentieth century described an inherent lack of predictability in a system of three or more interacting bodies: a tiny error in initial conditions could potentially cause a drastic change in the system. The concept, known as “sensitive dependence on initial conditions”, was described by Edward Lorentz, known as the first experimenter in the area of chaos in the early 1960s (he was working on the problem of weather prediction).

Chaos theory may be described as sensitive dependence on initial conditions. Just a small change in the initial conditions (for example, an error in a measurement or a rounded number) can create a huge change in the long-term behavior of a system. For example, starting with the number 2, the final result of a process could be entirely different from the same system with a starting number of 2.000001.

Erratic behavior appears to be random, but is not. The essence of chaos theory is a search for underlying patterns of a kind that have been discovered in a wide variety of seemingly random systems and that can (usually) pass all statistical tests.

For given condition x_0, and control parameters k, within domain R and given the impossibility of measuring a variable to absolute accuracy (such as sampling error or measurement error), small—even tiny—perturbations can lead to chaos. This means that given an open neighborhood of x_0 with radius ε:$U(x_0,\varepsilon)$, $k \in R$, there exist $n \in N$ (integers) such that if any $x^*_0 \in U(x_0,\varepsilon)$ (i.e. $|x^*_0-x_0|<\varepsilon$), $|T1(x_0)-T2(x^*_0)| > n$.

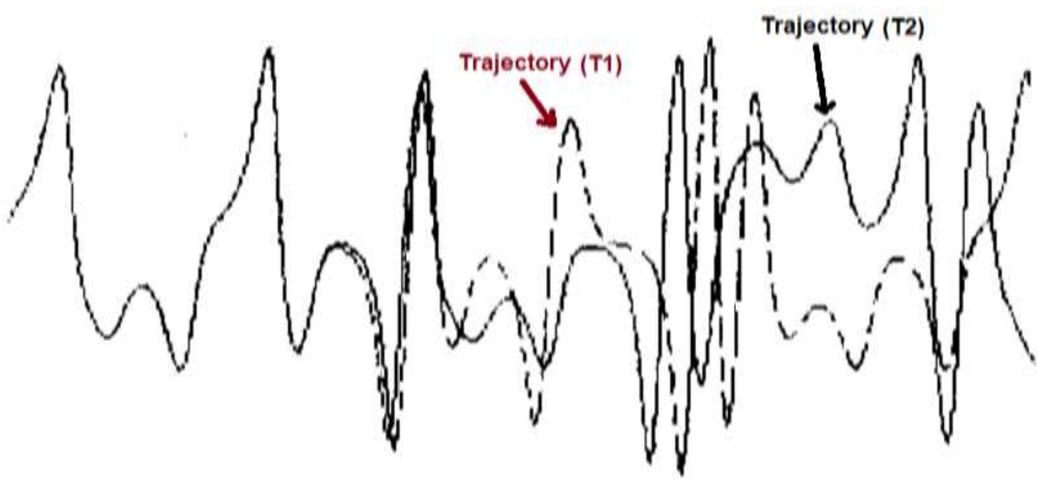


Figure 1: Lorentz’s experiment: the difference between the starting values of these curves is only 0.000127 [13]

Thus, forecasts of long-term behavior are meaningless.

A “local Lyapunov exponent” evaluates trajectory separation over short time periods (i.e. over local regions of the phase space). Local Lyapunov exponents can be useful in prediction over short time scales and in assessing how accurate a forecast might be. The idea of “information” is vital in chaos theory. As long as there is some relation between two variables, one contains information about the other. It happens in every feedback system; past events affect today’s events, and today’s events affect the future’s events.

There are nonlinear equations of the discrete times and continuous categories (differential equations) that can lead to chaos. The most frequently used example of chaos phenomenon is the logistic equation [14,15]: $P_{n+1} = kP_n(1-P_n)$.

Where, k is a control parameter in the equation of the discrete times. Despite its simplicity, the logistic equation has been shown to be the basis of chaotic dynamic behavior in a wide variety of chaotic nonlinear systems. Iteration of the equation will eventually demonstrate a discernible pattern.

Biologist Robert May has used the equation to predict population of species over successive time periods. In May’s predictions, P_{n+1} is the population for next year. It depends on the population for this year P_n. The population is a number between 0 and 1, with 1 representing the maximum population and 0 representing extinction. k is the growth rate, $k \in [0,4]$. Starting with a fixed value k and an initial value P_0, one can run the equation recursively to obtain P_1, P_2, …, P_n.

When k<3, P_n converges to a single number. For example, k=2.7, P_n converges to 0.6292. When $3 \le k \le 3.57$, P_n no longer converges to a fixed value; it oscillates between two values. This means that a species population would be one value for one year and another for the next year, then repeat the same cycle forever. If k is increasing a little more, P_n converges to a four, and then eight values, and so on. See the bifurcation diagram below:

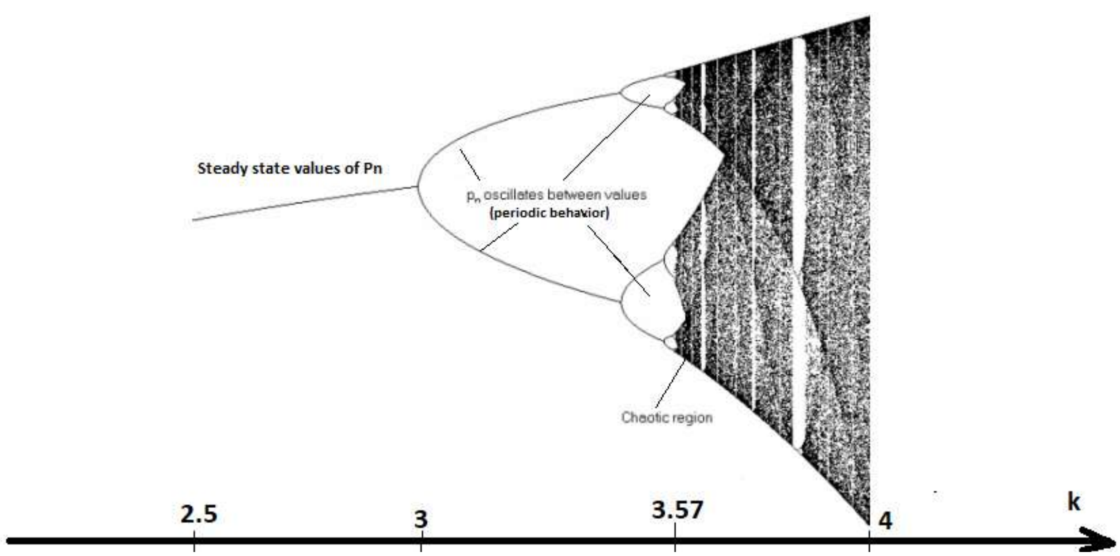


Figure 2: The bifurcation (period-doubling) diagram for the population equation

Physicist Mitchell Feigenbaum discovered the exact scale 4.669 at which a self-similar double is produced and 2.5029 for the fork-width scaling law [16, Chap.12].

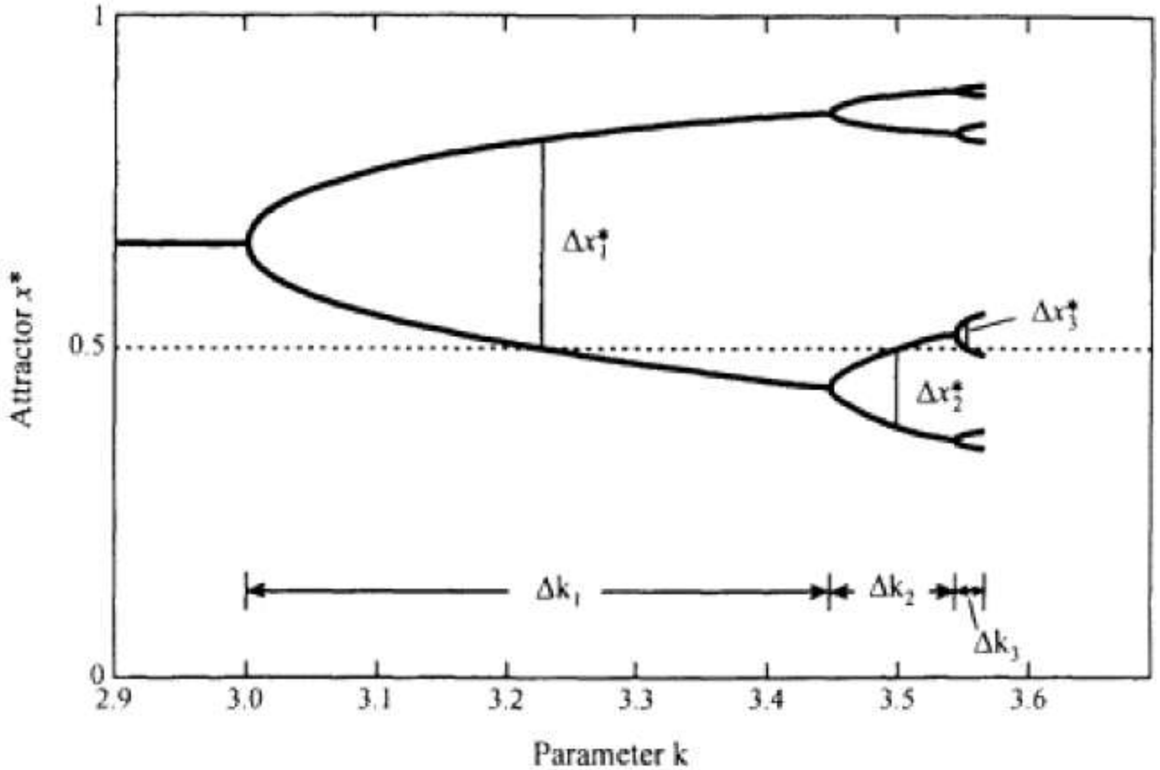


Figure 3: Period-doubling with the logistic equation
and attributes for scaling laws

Here,

$$\lim_{n\to\infty} \frac{\Delta k_i}{\Delta k_{i+1}} = 4.6692\ldots \quad \textbf{and} \quad \lim_{n\to\infty} \frac{\Delta x_i^*}{\Delta x_{i+1}^*} = 2.5029\ldots.$$

where n is the period number.

Observe that the bifurcations come faster and faster. This is the period-doubling road to chaos. When k=3.57, P_n becomes completely random. For values of k >3.57, chaos appears. The white strips and black strips in the figure indicate the chaotic regions (Fig. 0.4).

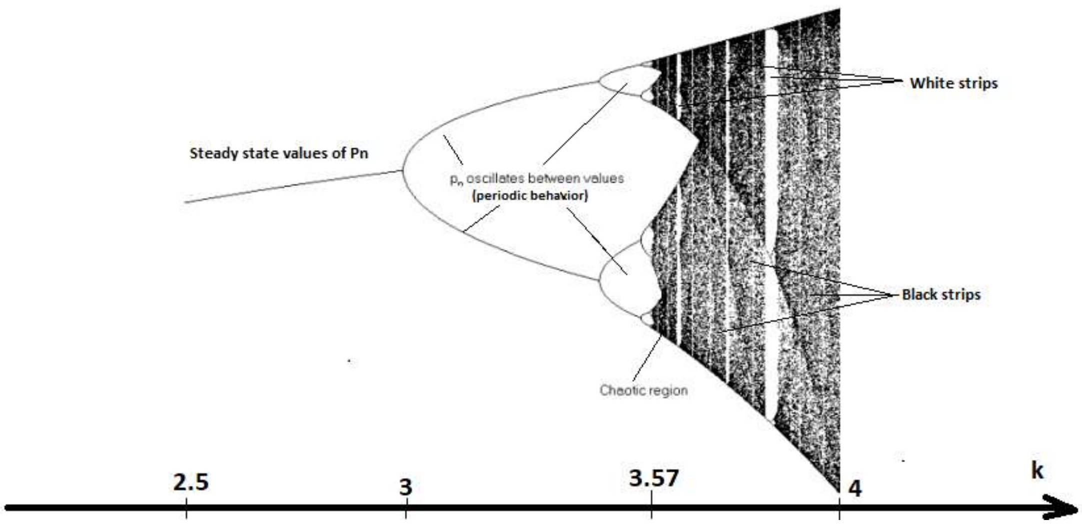


Figure 4: Note the white and black strips in the chaotic region

Looking closer at these strips reveals where the bifurcations appeared before returning to chaos. Thus, even when systems are totally chaotic, there are some motions in the system, both periodic and non-periodic, that occur over and over.

A chaotic system for creating trends and patterns

Only two kinds of equilibrium were previously known: a steady state, in which the variables never change, and periodic behavior, in which the system goes into a loop, repeating itself indefinitely. They appear between regular and chaotic regions. Lorentz found a third one. He called it a “strange attractor”, now referred to as a Lorentz attractor, chaotic attractor, or fractal attractor (Fig. 0.5). It appears in a chaotic region. The attractor is an extended concept from general equilibrium theory; an attractor is an equilibrium level of a system, or the level that a system reverts to after the effects of perturbation on the system fade away. A chaotic attractor is a set of possible different values that a system converges around. This set is infinite in numbers but limited in range, and not periodic.

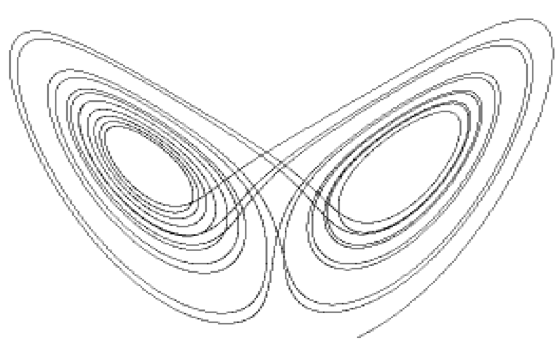


Figure 5: The Lorenz (strange) attractor

A steady state and periodic state are a point attractor, and periodic attractor or limit cycle, respectively. They are stable and appear in non-chaotic regions. On the other hand, with a chaotic attractor, the trajectories plotted in the phase space of a system (a graph of velocity versus position) never intersect, although they wander around the same area of phase space. Orbits are always different, but remain within the same area; they are attracted to a space but never converge to a specific point. Cycles, while they exist, are non-periodic. Equilibrium applies to a region, rather than a particular point or orbit; equilibrium becomes unstable or dynamic.

The phase space trajectory may have fractal properties (we'll discuss fractals below). The following two criteria for the chaotic attractor are 1) sensitive dependence on initial conditions and 2) a fractal dimension.

The fourth order of chaos is zones of relatively greater popularity on each chaotic attractor. These are zones in a chaotic system more likely to be visited during its evolution (the black strips in Fig. 0.4). This structure provides some guidance in devising a statistical theory for accurately predicting the likelihood of x taking on a particular value.

Fractals

A fifth orderly feature of chaos is the fractal structure [18]. The fractal is a line, surface, or a pattern that looks the same over a wide range of scales. In other words, a fractal is self-similar, and scale invariant.

The Sierpi'nski sieve is an example. To create a Sierpi'nski sieve, add inside the original triangle a new white triangle. To complete the shape, the procedure is repeated indefinitely. A magnification of the Sierpi'nski sieve looks the same as the original. It is self-similar.

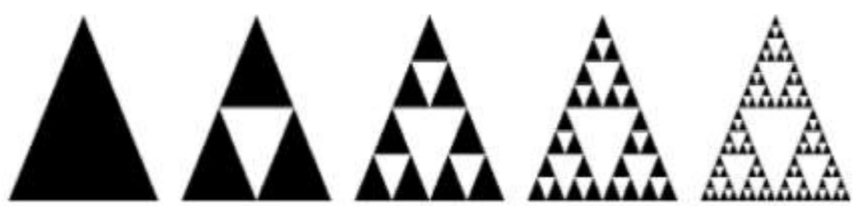


Figure 6: The procedure and complete Sierpi'nski sieve.

The Koch curve is another example of a self-similar phenomenon. To create a Koch curve, to the middle third of each side, add another equilateral triangle. To complete the shape, the procedure is repeated indefinitely.

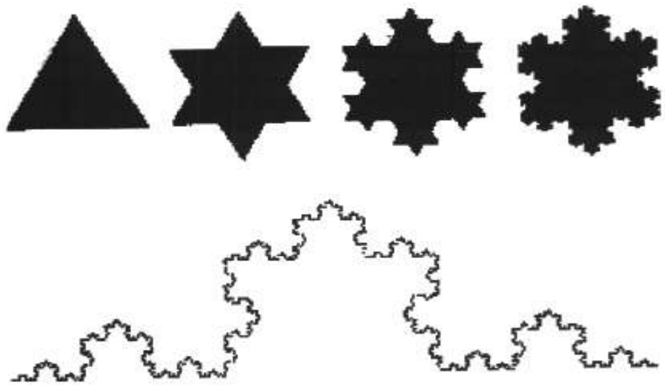


Figure 6: The Koch curve, first four iterations and eventual curve

Fractals by definition are rough and jagged instead of smooth. A planar fractal is rougher than a smooth curve line, which has one dimension. A fractal also does not fill up space in the same way as a square or circle, which has two dimensions (Fig. 0.6, 0.7). So, its dimension is a non-integer ϵ (1,2).

Figure 7: a fractal chart is not fit with smooth curve line or not filling up a square

The Lorentz attractor is a fractal; the bifurcation diagram of the logistic is a fractal.

The mathematician Benoit Mandelbrot studied cotton prices, and price distributions that seemed aberrant in short-term analysis, but the *degree* of variation remained constant: at scale, there is symmetry. Consider the Koch curve, mathematician Helge von Koch's paradoxical fractal, which has an infinite perimeter (because each iteration increases the length of the curve by a factor of 4/3), but encloses a finite space. The iterations of fractals can be thought of as the "bumpy", ever-changing price line, but the regularity of the pattern is immutable.

The two main types of fractals are deterministic and natural fractals. Deterministic fractals are generated by iterating an equation. Therefore, they are exact. For example, the Mandelbrot set is defined by the equation $z = z^2 + c$. Shishikura (1994) proved that the boundary of the Mandelbrot set is a fractal [19]. Natural fractals simulate natural objects such as clouds, rocks, noise, landscapes, and so on. Therefore, they are just approximately similar under change of scale.

In a chaotic system, surprises occur when the individual components are added together. This is a special class of a process called self-organization. Self-organization occurs when a self-propagating system, without outside influence, reflects a tendency for a dynamical (changing over time) system to organize itself into more complex structures. The structure varies from one case to another during its evolution. For example, the organizing of weather elements (wind, moisture, and so on) into hurricanes, of water molecules into laminar flow, and of the demand for goods, services, labor, and so on into economic elements. A self-organizing system reverts to an equilibrium level or symmetry of a system after the effects of perturbation of the system fade away.

The financial markets (FM)

(Note on terms used in this paper: "Chart" refers to a chart of a price of a security or rate of a pair of currencies. Price or rate is noted as "P".)

There has been discussion of whether FM are random, cyclical or both. Some people believe the markets are efficient (EM). The others believe there is a certain rhythm to the markets for which both fundamental and technical analysis can help gain an insight advantage, i.e. Inefficient Market (IEM) [8]. Moreover, there also exists evidence of IEM in nonlinear and chaos phenomena [2].

The Efficient Market hypothesis (EMH) holds that it is impossible to "beat the market" because all relevant information is already reflected in a given issue's price/rate and in the market as a whole. According to EMH:

The stock market embodies turbulence, mayhem, and unpredictability.

Price fluctuation is purely random.

The market has no memory.

Ups and downs on any given day bear no relationship to previous moves.

No real economic trends occur.

There is no underlying dynamic.

Any extreme event such as a drop of more than 500 points a day is considered to be an accident [9].

Economists use random walk or Brownian motion to describe efficient behaviors of the financial markets [10, 11].

On the other hand, the Inefficient Market (IEM) is one in which an asset's price doesn't accurately reflect its true (fair) value. Market inefficiencies exist due to information asymmetries, transaction costs, market psychology, and human emotion, among other reasons [8,9].

In fact, many traders in the FM today believe there is no pattern in the price of securities. Price action is considered to be random, and large moves are a reaction to High-Frequency Trading (HFT) [10], recession, pandemic, etc. Overreactions push prices up or down to make an unbalance or asymmetry of FM. This leads to volatility in FM. Longer trends like a strong run-up in FM show that an effective model must have some memory of past behavior.

As just one example of the financial markets responding to human emotion: Tesla lost money for 14 years, and showed its first profit in 2020, but for a long time its stock price was highly inflated due to—what? A widespread rational belief in the bright future of EVs? Or—fandom?

As a result of illogical forces such as emotional enthusiasm leading to fandom, or doomsaying with no foundation, some securities may be overvalued or undervalued in the market, creating opportunities for profit. Living in an IEM world certainly undermines economic theory, and in particular the efficient market hypothesis, but an IEM world offers a wealth of opportunity. Knot theory offers an intriguing way to approach the market.

Traders and investors who follow EMH know well the unpredictability of the stock market. A chaotic attractor exists for the S&P 500 [2]. That is, an underlying, non-periodic attractor could explain movements in FM. Some researchers believe that chaos theory may be appropriate to FM, and some contend that the market may be becoming unbalanced or asymmetrical, perhaps leading to volatility of a kind never seen before [9].

Real causes and effects do govern market behavior. An asset's price today does influence its price tomorrow. Thus, financial price series may be said to have memory.

Because of the sensitivity to small perturbations mentioned above, just a little instability can rapidly devolve into large-scale booms and busts. Chaos provides a natural way of seeing the crucial connections between the details of trading and large-scale dynamics of the market. Large-scale movements in the financial markets arise from millions of individual decisions to buy and sell [9].

For example, one can consider a given down pattern to see whether it is sensitive to small perturbations if its closed price (point S) falls below closed price of the previous candlestick (point B) in a certain time frame (TF). This could be because of a measurement error due to the impossibility of measuring a variable to absolute accuracy. This discrepancy might in itself trigger a sell signal and begin a down trend by a psychological rule: the crowd follows the crowd.

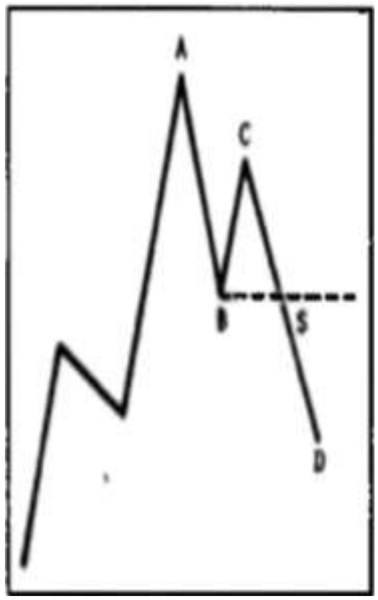


Figure 8: A down pattern

This also was the pattern for the crash of FM in 1987.

Figure 9: The crashing pattern for crashing of FM in 1987

There is a balance between buyers and sellers while the market is sideways; otherwise, there is an imbalance when certain factors are in play. There are “only sellers” while a market is sold off by a recession, war, pandemic, and so on.

There exist patterns of the chart such as Head & Shoulder, double bottom, knot, etc. They repeat and scale independence by fractal property of the market. When there are chaotic leaps in price, the price changes discontinuously, in jagged steps. In the traditional way of economic thinking (EMH), smoothing out the bumps and jags of FM’s data such as calculating the “moving average” to remove randomness or noise from the real information is an approximation of the theory of distributions or generalized functions [20, Chap. 6]. However, working within the theory of IEM, there is another way that gives us more information.

Knots

In a previous paper [4], the author has shown how to find a knot in the chart, and some its properties. The following explains more properties of knots of the chart.

The trefoil knot

Figure 10: A trefoil knot of AUD/USD at 15m-chart.

Link

A link (L) is a union of knots (K_i), i=1..n. It is shown thus: $L = K_1 \cup K_2 \cup \ldots \cup K_n$

A link of USD/CAD at daily-chart was forecast in May 2019 [21]. The missing point here was III (Fig. 11).

Figure 11: A link of USD/CAD was predicted.

Here was the reality of the scene later on (Fig. 12). The link performed utterly as predicted by the previous chart.

Figure 12: A link of USD/CAD

Dual knot

A dual knot is the knot created in a down trend. For example, the knot K_2 in Fig.11 is a dual knot. Thus, the trefoil knots and their links have been found in the chart at different time frames, demonstrating the fractal property of iteration.

An error

An unknot could be added by Knot + unknot = Knot (see Fig. 13). It could be considered an error and this is explained below.

Figure 13: An unknot is considered an error of a knot.

The unknot is added to the chart by randomness of the market, as price is going back to a certain point to perform a certain symmetry, or to complete a pattern. This is also a property of fractals. Knots in reality are similar but not exactly the same.

A crossing point

If price/rate is crossing back to a certain point, we can call this a back point (BP). For example, the point A in Fig.14 is a back point (BP). Coming back to a BP to create a crossing point makes a cycle (see Fig. 14).

Figure 14: A crossing point (AB) was created in chart GBP/USD in a 4h-chart

Looking at crossing points offers another way to discover the knots. This topic shall further be discussed.

Sideways markets

A sideways market occurs when the price of a security oscillates in a horizontal range without forming any identifiable trends over a period of time. Sideways movement can take several different forms.

Sine-form sideways

Figure 15: The Bitcoin/USD is in a sine-form sideways

The knot structure of the sideways was presented in [5] with sine form.

Wild-knot-form sideways

Wild knots are the result of a process of infinite knotting: the twists of the curve get smaller and smaller and converge toward a limit point as a wild point of the curve [7].

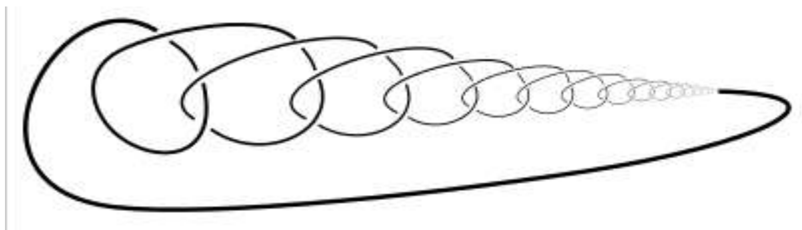


Figure 16: A wild knot

Figure 17: A wild-knot-form sideways.

The price will break out or break down stronger (in a candle chart the candles will have long bodies) with a wild knot than with a sideways sine form.

Chart rotation

When a trend is emerging, the chart rotates while it changes the trend. Hence, the patterns also rotate (Fig. 18):

Figure 18: Sideways or Consolidation in rotation of the chart while it changes the trend.

Moreover, a sideways structure can be represented by the trefoil knot (Fig. 15):

Figure 19: A trefoil knot structure of sideways movement of Bitcoin (Fig. 15)

Internal symmetry of knot forms

Knots are chaotic attractors, and price movements may be seen to occur in a lattice of hidden chaotic attractors. The market imitates nature in that it has a certain symmetry. The patterns of the chart contain information about their innate symmetry which, when discovered, can predict price action. Forecasts of long-term or short-term trends depend on a certain Time Frame (TF) of the chart. The internal symmetry of the trefoil knot is key to understanding forecast by symmetry.

The internal symmetry of the trefoil knot (note the directional arrows) is defined as given the property 3->1, it also has 1->3 (31,13). Thus, we have the following symmetric pairs: (31, 13) , (12, 21), (23, 32). (See Fig. 20)

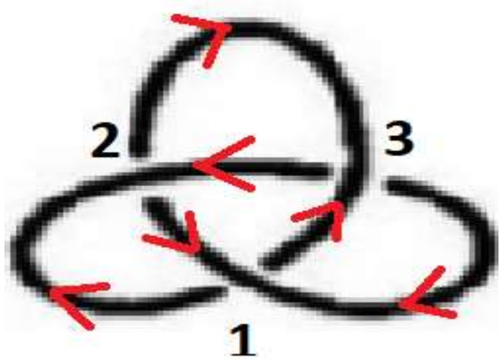


Fig. 20: The symmetry of the trefoil knot

The symmetry is rather like the symmetry of a three-lobed leaf [23] or Mirror Symmetry (MS) where MS (left side of leaf)=right side of leaf (see Fig. 21),

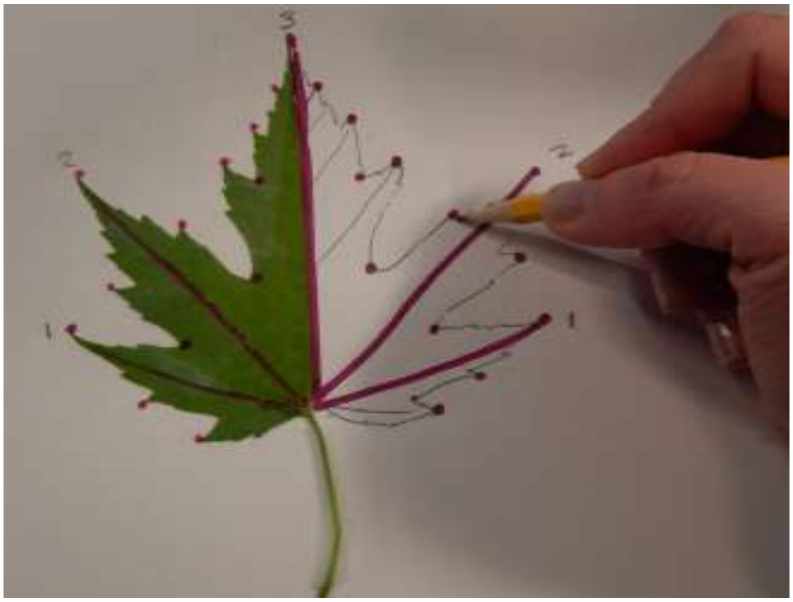


Figure 21: The mirror symmetry of a leaf.

The leaf symmetries in the chart below are apparent:

Figure 22: AUD/NZD has a leaf symmetry

If the symmetry of the chart is broken, there is a mechanism fixing the asymmetry to the symmetry of the chart. This mechanism needs the figure-eight knot to perform.

A 8-figure knot in a price chart

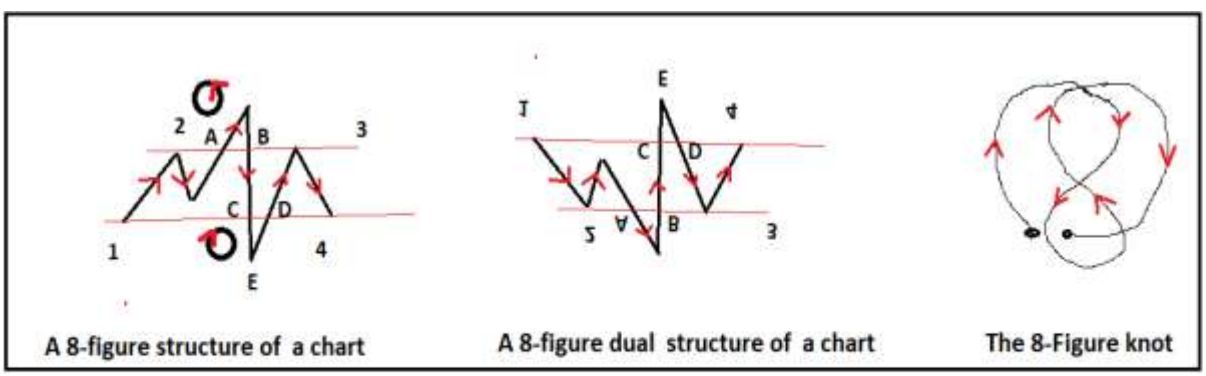


Figure 23: Recognition of an 8-figure knot in chart.

Fixing asymmetry to symmetry indicates an 8-figure knot as the following:

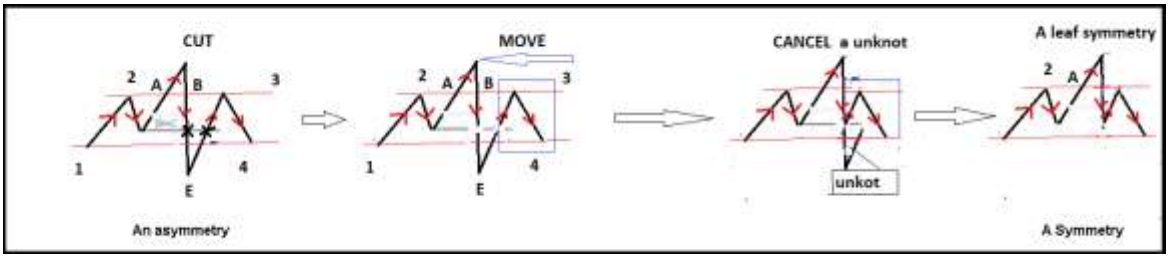


Figure 24: The 8-figure knot appears to resolve an asymmetry to a symmetry.

Figure 25: Fixing an asymmetries to the symmetries of BTC/USD in a 30m chart

The symmetries above are broken. We have the following pairs:

- For 8-figure knot: (aa',bb');(cc',dd')
- For dual 8-figure knot: (ee',ff'); (gg',hh')

Thus, if the symmetry of the chart is broken, FM is moving such that its chart resumes its initial symmetry. This is a property of self-organization of the trading system, like the natural ecosystem [17]. For example, someone disturbs the flat surface of a lake by throwing in a stone. Waves travel to the shore of the lake. The surface of the lake returns, in due course, to its initial balanced, symmetrical state.

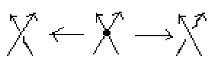
The self-organization is an important and self-evident property of nature. Charts also have the self-organizing principle. Thus, the trading system is a complex and self-organizing system. The self-organization property in a financial market chart can make it look as though the chart has a memory, with prices moving back to previous asymmetric points. This means if price breaks out, it will break down later, and vice versa (see Figs. 19, 25).

Asymmetries in small TF are hidden in large TF in the time evolution of a chart. Fixing asymmetry to symmetry occurs in different time frames by the fractal property (scale independence) of the chart. Knots and their properties are independent of time frames.

We explain these by the knot theory of the chart [4,5]. The chart is a space Ғ of singular knots. The crossing point is a double point or a singular point. Singular knots have a finite number of ordinary double points.

A finite-order Vassiliev invariant is a function υ: Ғ -> R. Each double-point in the singular knot satisfies the following relation:

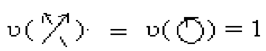
The Vassiliev invariant elucidates the changing directionality of the knot as it cycles:

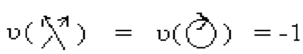


N_+ N_- .

N_- is derived from N_+ by the mirror symmetry. i.e. $MS(N_+) = N_-$

By nature, the directionality of the ordinary knots is cyclical. Hence, we have the following normalization:





A cycling ordinary knot means if we are moving either forward or backward, any point on the knot will come back to its starting position.

The Ғ's partition into n different subsets Σ_n n=0,1,2,3,.., by Vassiliev finite type invariant. The singular knots in the same subset are equivalent.

$Ғ \sqsupset \Sigma_0 \cup \Sigma_1 \cup \ldots \cup \Sigma n$

Here, Σ_0 the set of ordinary knots and by Σ_n the set of singular knots with n-double points.

A singular point is a point by definition at which the price will break out or break down later [5]. The self-organization will smooth out or cancel the singular point. The Vassiliev invariant gives a difference of two states of the singular point. The difference between the two states is their orientations (break out or break down). Thus, the singular points are equivalences, i.e. $MS(N_+) = N_-$. But, these just are approximations in reality. They are differences by degree of randomness and fractal of the chart. In general, $MS(N_+) = N_-$ + unknots + fractals (Fig. 25). It has been proved that $\upsilon(K_n) = 2^n$, $K_n \in \Sigma_n$ n=0,1,2,3,.., We ignore unknots while we count the structure of a certain knot in the chart (see Fig. 1.4) because of the property $\upsilon(K_n U K_0) = \upsilon(K_n)$, $K_n \in \Sigma_n$ n=0,1,2,3,..,

The origin of the symmetry in trading is a game of zero sum. It means that Σ (profit+loss)=0, for every trader and investor. This is a conservative law in trading and points to a symmetry in the chart. If the chart is in a consolidation state (sideways), there is a balance between buyers and sellers (Fig. 15). If unbalance or asymmetry trades $\Sigma \neq 0$ at a certain time (for example, traders and investors sold their holdings much more than bought during some panic time such as COVID-19, a recession, war, or HFT could also cause this [10]), FM is moving so that its chart resumes its initial symmetry. Hence, we have a trade balance.

Recognizing market symmetry is part of having a wide analytical perspective of the market of your choice. The more you can do to expand your perspective, the better prepared you are to take advantage of opportunities before they are apparent to the crowd.

The Fourier series of the knot

The knot can be represented by a certain polynomial [21], a mathematical abstraction. The knot is demonstrated in the chart below (Fig. 26) and one can see at a glance that the chart has cycles and trends. It can be represented by a Fourier series. Fourier analysis is a mathematical tool for describing a time series of periodic constituents as sine waves. Thus, a knot → a Fourier series → a function → a polynomial (Taylor's series expansion). Hence, a real relationship between a knot and a polynomial is established.

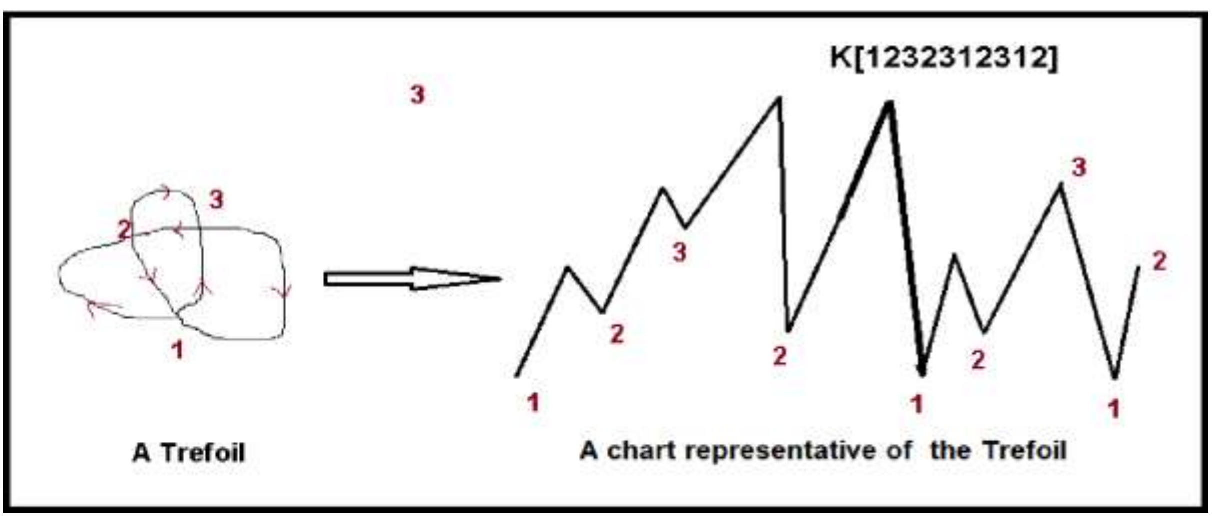


Figure 26: A representative of the trefoil knot in chart

Thus, the existing knots in the financial chart are evident for the time series that has periodic and non periodic structures. A random time-series doesn't have any periodicity.

In addition, knowing a time series is periodic is important information that suggests how to proceed in chaos analysis and reveals auto-correlation. For example, if the return to a price at time t+1 depends on the return at the time t ($R(t+1)=f(R(t))$, the return is said to be auto-correlated. If the chart (the time series) of a price is the trending regime, its return is positively correlated.

Some Theorems

(Note: The price of a security S or a rate R of a pair of currencies is noted P. We assume that a good S/R exists for the long-term.)

Theorem 1

P is repeated at least twice.

Proof

P is given any. There is an ordinary knot K in the chart so that P belongs to K (the chart is a space of knots). Hence, P is repeated at least twice because K has the cycling property [5]. QED.

In fact, P could repeat several times (Fig. 27)

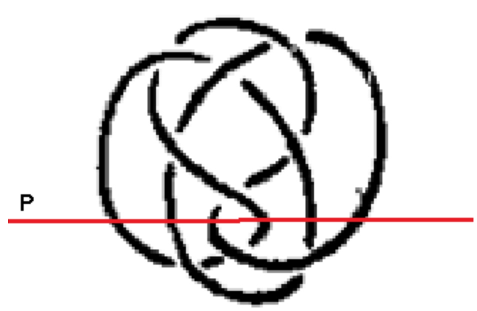


Figure 27: A knot with several crossing points. Price P repeats 6 times.

Theorem 2

Return of P without fee is positive or zero.

Proof

P is given any. There is a knot K such that $P \in K$. Get $P_1 \in K$ and $P_1 \geq P$. P_1 repeats at least twice by theorem 1. Hence, the return of $P = (P_1-P)/P$ + return of dividend (d) ≥ 0. QED.

The result is more general for different asset classes than formulas 72 [24]. Formula 72 is calculated based on the projected yield of dividends of a security. Thus, we could invest in good firms with dividends for the long term (as in Warren Buffett's manifestly successful strategy), without diversification in different asset classes according to modern portfolio theory [25]. This theory is in fact based on EMH.

When to go long-term

Some cautions are in order about holding issues long-term:

a. Technology stocks will be subject to major price swings for the foreseeable (long-term) future, largely because technology changes fast, with major players replaced by other major players in a much swifter turnaround than was common in major companies of, say, the majority of the past century. Google is the big kid now, but once it was Yahoo, and, incredible to think of now, Netscape Navigator.

b. Bank stocks also, and other cyclic stocks, are especially vulnerable in economic downturns and can go bankrupt, as Washington Mutual Bank did in 2009.

c. In an economic recession, usually the U.S. Federal government will launch a bond-buying program, referred to as Quantitative Easing (QE), to maintain a low interest rate and to stimulate the economy. This is one factor that sends the stock market back up [26, 27].

Buying and holding, riding out the downturns, most often works in the investor's favor. Consider the chart of SPY at the time of the COVID-19 pandemic outbreak in March 2020 (Fig. 2.2). SPY is an ETF of the market index S&P 500. It was down 32.65% in the second quarter of 2020. However, SPY recovered its losses and reached its previous high after fewer than six months. At the start of the COVID pandemic, the end of March 2020, the SP&500 dropped around 34%. It began a bullish market in April and finished more than 16% higher at the end of 2020 [10, 21, 22].

Figure 28: The chart of SPY

If an investor does not hold P_1 long-term, no profit is gained. Money management is important so that you don't sell at a loss when you need money while the market is in a down trend. In addition, you should manage your funds so you can take advantage of every opportunity to buy or add more shares while stocks are at discount prices.

At the end of March 2020, the S&P 500 dropped around 34%. It began a bullish market in April and finished more than 16% higher at the end of 2020 [28].

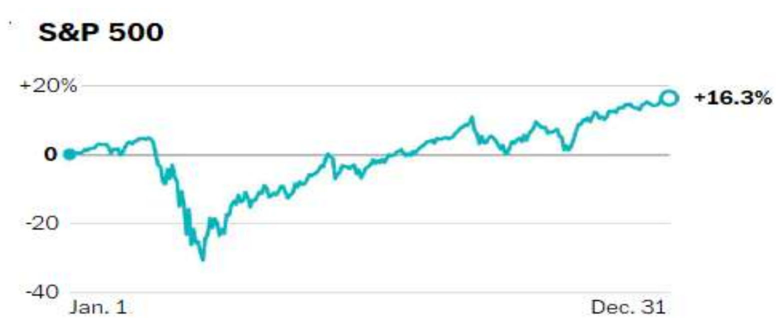


Figure 29: The S&P 500 at the end of 2020

d. I recommend against trading contract for difference (CFD) on volatile issues such as FX, because rollover fees on the price spread can eat up your profits if you play swing or long-term.

e. Trading bots are not optimal because a tiny error in closing price can change trading results. The back test is reliable, though, because the closing price is known.

f. Much would be simplified by a trading system connecting all of the global trading platforms.

Scales of collapse

Extreme volatility of the financial markets is currently loosely defined as a drop of more than 500 points in a day, while the FM effect of a recession, pandemic or war is too vast to be explained in terms of random fluctuation. However, these phenomena can be illuminated using chaos theory and specifically, knot theory.

a. The crowd follows the crowd.

b. Price moves follow a diagram of a certain knot K (see Fig. 30).

c. Note that a knot is a chaotic attractor in the phase space, using the method in [3]. The chaotic attractor actually explains most movement in the market.

d. Price is going to break out/break down after a knot or a link is performed. It could be going chaos, coming back and moving around the knot.

Consider the following examples:

Figure 30: A chaotic attractor (a trefoil knot of NZD/USD at 27-29 of Jan at H1)

Figure 31: Price/rate of NZD/USD was moving around the attractor.
The attractor was unseen in larger TF.

Figure 32: Price was moving around the chaotic attractor (the Knot of BTC/USD (Fig. 18))

Figure 33: Price was moving around the chaotic attractor-The link of USD/CAD (Fig. 12).

Figure 34: Price was moving around the chaotic attractor-the knot of AUD/USD. (Fig. 10)

The Cycles and Trends

From the point of view of symmetry, we can explain cycles and trends in FM. Knot and unknot are cycles. The price/rate moves follow the diagram of the knot to create trends and cycles. Moreover, the knot is also the chaotic attractor in the phase space. The attractors play a role in support and resistance levels. The price/rate responds to the attractors and moves around them. Price/rate retreats when it closes in on the chaotic attractors, and advances when it gets far away from them. They are evidence for reversed mean in FM.

Entropy of the chaotic attractors

Chaotic attractors are created more in evolution of the price in time. Thus, entropy of attractors increases with time. The attractors are hiding and become normal points of charts in larger time frames such that we don't recognize them unless we are looking for them and noting them. This is one reason why many traders lose money with short-term trades. Both swing trading and long-term investment can be taken slower than before because the price collides with chaotic attractors much more in evolution of price in time.

Thus, there is a virtual networking of chaotic attractors. The price looks like a wave traveling in a network. Consider soliton (solitary) waves (Fig. 35, 36). (In physics, a soliton wave is a stable, self-reinforcing wave packet that propagates at a constant velocity [7]). They are sideways states in FM.

Figure 35: Soliton Waves of BTC/USD

Figure 36: Soliton waves of USD/CAD

Timing or forecasting

We could be timing the price/rate of the chart in the financial markets. If we have sufficient data, we can predict the long term. But, the scale of the prediction depends on how long we have been gathering data. The lack of data limits our future predictions. For example, we could predict 20 years for the pairs of currencies if we just have 20 year data. The following elements could be used for timing:

a. The trefoil knot of the chart.

b. The knot's cyclical property.

c. The crossing point (BP).

d. The market going chaos then returning to the chaotic attractors. For example, the trefoil knot of EUR/GBP at h4 occurred July-October 2021 (Fig. 13) and a daily chart in December 2021 (Fig. 37). Therefore, we can forecast its price will be back at the knot (the attractor) at the time that author wrote this section.

Figure 37: Forecast: EUR/GBP was going back the chaotic attractor

This also happened in September 2022 (Fig. 38)

Figure 38: EUR/GBP was going back the chaotic attractor.

The prices or rates of the charts of BTC/USD(Fig. 32) and USD/CAD(Fig. 33) were heading back to their chaotic attractors (the knots) also.

e. By symmetry of the knot that we can forecast long-term or short-term depends on the Time Frame of the chart in which we analyze it. Here, the author thinks that we could predict short-term, not long-term, in theory of chaos because we don't count on the symmetry of the chart–invariant against variant. Here, invariant is the symmetry.

f. Price/rate could go chaos after breakout/breakdown after the knot is performed. Thus, we could find out the duration of the price going into chaos by confirming when a certain knot is performed completely.

g. By soliton waves.

CONCLUSIONS

The financial markets, as rife with randomness as they are, contain elements of determination that can be identified and used reliably in many cases for prediction, if properly understood. Chaos theory and knot theory are applied here to the financial markets. The knot is seen to be a hidden order of chaos–the attractor in the phase space. It is the basis of market movements. It is sensitive to initial conditions and a fractal dimension. Within an appropriate domain, the trajectories are around the same area in phase space, giving us a new way to understand trends, cycles, and mean reversion of FM.

Extreme volatility of the market currently is defined as movements of more than 500 points a day, or down 10%. A 20% movement over a few days could be explained by price/rate could be going chaos after a knot performed completely or price/rate follows a certain knot diagram.

Thus, we have demonstrated two of the mechanisms that move the market. The knots (unknot and trefoil knot) are created in the chart to build up strength toward either breakout or breakdown of the chart later. The chart transitions to a new phase. The phase transition is a discontinuous or vertical motion to make the chart continuous or that reverses the trend [4]. It also is a basis for breaking the symmetry of the chart. If the symmetry of the chart is broken, the market will be moving so that its chart eventually resumes its symmetry. Moreover, the author has proved that long-term return of a security or rate without fee is proved ≥ 0. We have shown that price moves in a network of chaotic attractors (knots) in certain TF. The soliton waves of sideways price movement also are demonstrated.

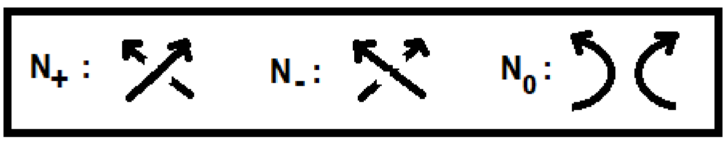
Unknot is considered as an error in the formula knot = knot + unknot. Thus, IEM has to have its corresponding money management system instead of EM's money management system [23]. This topic will be addressed in another article.

LIMITATIONS OF THE CURRENT STUDY

While an attempt has been made to show certain possibilities of diagrams in price charts that arise from different knots, obviously the sheer amount of data generated in a chart moving through time means that the diagrams shown here are only a tiny sample of the amount of information that is theoretically available when knot theory is applied to price charts. This is especially true when considering the fact that the orientation of a knot in a chart can give rise to a different diagram and interpretation.

The Vassiliev invariant, a finite type of knot invariant, gives a difference of two states of the singular point. The difference between the two states lies in their orientations in a price chart (indicating upward movement through resistance level or break out, or downward movement through support level or break down). They are states of the mirror symmetry (MS). However, it is important to understand that MS(break out) ≠ (break down) in reality. The invariants can come from 1) different structures of a certain knot (for example, Figs. 1, 2, and 4 show the same trefoil knots but have different diagrams) or 2) different structures of singular points of the knot.

If we define a general singular point of a knot, we get a set of three states [22, p. 67]:



Then a general invariant $\upsilon(K_n) = 3^n$ (n is a non-negative integer) is an approximation in reality, as in the case of the Vassiliev invariant. Recognizing the symmetry of the general singular points is a very interesting problem in forecasting price action. It may relate to the tricolorability of a knot (for example, the figure-eight knot is not tricolorable).

There are many knots with different crossings [22, p. 7], and it is impossible to identify all of them in a chart without the data processing power of computers. The same knot can have different diagrams in the chart according to up/down orientation; in other words, the topology of the same knot may be represented by many different diagrams in a given chart. Storing all the possibilities in a database, possibly training AI to discover them in historic charts and apply that data to current charts, would facilitate research into the distribution and nature of chaotic attractors in time.

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