



## DYNAMICS OF COVID-19 PROPAGATION THROUGH NUMERICAL SIMULATIONS OF MATHEMATICAL MODELS

**Pranvera Mulla** 

Faculty of Natural Sciences, Department of Applied Mathematics,  
University of Tirana, Branch Sarande, Sarande, Albania  
[pranvera.mulla@unitir.edu.al](mailto:pranvera.mulla@unitir.edu.al)

**Eglantina Xhaja**

Faculty of Natural Sciences, Department of Applied Mathematics,  
University of Tirana, Tirane, Albania  
[eglantina.xhaja@fshn.edu.al](mailto:eglantina.xhaja@fshn.edu.al)

### Abstract

*For Albania, the spread of COVID-19 has created social and economic challenges. The purpose of this paper is to describe some of the mathematical models that give the most reliable projections which are closer to the epidemiological reality. The fact that the data are not always complete shoës the importance of using mathematical models, which since they are based on differential equations, give a better representation of the dynamics of virus spread.. The model used SIR (Kermack and McKendrick, 1927), has the advantage as its parameters can be adapted to the arrival of new data and the predictions made are as close to reality as possible (Cooper et al., 2020). Through numerical simulations, this paper also shows the effectiveness of social measures taken by governments. Since the accuracy of the model prediction also depends on the quality of the data, it is important that the data collection process is improved not only in terms of the number of tests conducted, but also in terms of the qualitative description as well as the quantification of the adherence to the social measures by the population*

*Keywords: COVID 19, SIR model, numerical simulations*

## INTRODUCTION

The spread of the COVID-19 virus posed previously unimaginable economic and social challenges to Albania and other countries. Its purpose for the virus is complete in Wuhan, scientists under the great pressure of time and can present the study of the study for the prediction of the virus, enabling you not to help policy-makers, for the measures to be taken. Mathematical modeling performs part of the assessment and control of the recent outbreak of coronavirus disease 2019 (COVID-19). In this epidemic, most of the imposition of the tough enterprise measure for entering into COVID-19 compliance. The best question for governments is when and how decisions should be made to control COVID-19 as they have great social and economic impact.

The most sought-after solution is based on scientific evidence, whether it depends on data and models. One of the most critical problems throughout crises is whether it can be reliably developed epidemiologically to predict the evolution of the virus and to evaluate the effectiveness of various measures.

Statistical projections and forecasts are two categories which are applied for different purposes. According to Chimmula & al. (2020), projections are determinative and they explain what can happen under a series of basic hypotheses, while statistical forecasts use observed data to predict what will happen. Quantification of control and disease strategies Rachah et al. (2018) and its smoothing, is aided by models ranging from the classical SIR model to the more complex Ndairoua et al. (2018).

It is imperative that mathematical models be developed to provide knowledge and make predictions about the pandemic, to plan effective control strategies and policies (Scarpino, 2019, Rüdiger (2020).

Based on the theoretical framework of the SIR model, Cooper & al. (2020) have concluded that the spread of COVID19 can be controlled in all considered communities, if there are appropriate restrictions and strong policies to control infection rates since the onset of the disease.

Over the past year, various simple and modified SIR models have been developed to predict the course of COVID-19; Anastassopoulou & al. (2020) provided a preliminary estimate of fatality and recovery ratios for the population from a discrete-time SIR model.

Seman (2020) has studied virus spread from an interdisciplinary perspective, providing a better understanding of mathematical modeling for epidemic diseases Brauer (2012). Similarly, using a SEIR (Susceptible Exposed- Infectious- Recovery) framework, Lin et al (2020) predicted the effect of government and individual policies on the spread of the epidemic in China.

Contreras (2020), based on the SIR model, dividing the population into several levels representing S (sensitive), E (exposed), I (infected), A (asymptomatic), R (recovered), D (dead), Q (in quarantine). Ndaïrou & al. (2020), divide the population into eight levels by finding different combinations of variables.

Researchers around the world have performed numerous mathematical modeling and Numerical analysis on COVID-19 since its outbreak. In general, epidemics follow a similar trend, which can be mathematically modeled by Wang & al. (2020).

According to Roda et al. (2020) given the same database of confirmed cases in Wuhan City, more complex models may not necessarily be more reliable in making predictions due to the larger number of parameters of the model to be evaluated compared to a simpler model.

The predictive power of mathematical models Sameni (2020), is limited by the accuracy of the available data Carletti et al. (2020) and by the level of abstraction used to model the problem. Ivorra et al. (2020), shows that the use of models is also influenced by not taking into account some variables related to population distribution, the effect of humidity and temperature on SARS-CoV-2, official data reported, the quality of which is not known if it is correct.

## THE SIR MODEL

Building a dynamic predictive model that incorporates the dynamics of the Covid-19 epidemic to better understand the factors influencing disease transmission is an attempt to control the spread of the virus.

The original mathematical description of the spread of an infectious disease in a population is the so-called SIR model, due to Kermack and McKendrick (1927) which divides the (fixed) population of  $N$  individuals into three states:

$S(t)$  the number of individuals susceptible but not yet infected with the disease;

$I(t)$  the number of infected individuals;

and  $R(t)$  the number of individuals removed (recovered) from the infected group, either by becoming healthy again either longterm immunity or by passing away.

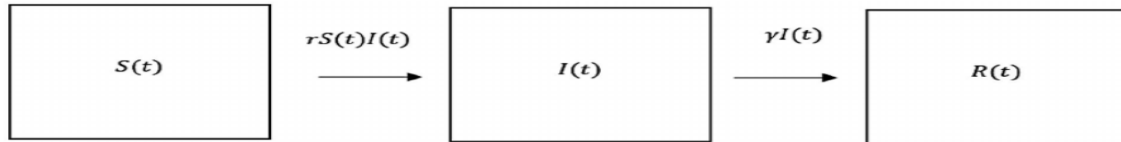
SIR models assume that vulnerable individuals contract the infection by acting on infected persons. Thomas et al. (2020) The term that models this interaction is represented as part of the product of the affected and infected:  $r S(t) I(t)$ . The main feature of projection models like the Kermack-McKendrick system is that they are based on logical assumptions of the basic mechanics of a process. It is assumed that the virus is generally contracted only once and an infected individual dies or recovers.

SIR models are sensitive to the basic assumptions of the model. When these assumptions are modified, forecasts sometimes change dramatically. Forecast models are most

useful once data has been collected. For example, most SIR COVID-19 models use fixed parameters, which lead to a constant called  $R_0$ . The base number  $R_0 = \frac{\beta}{\gamma}$  is an essential indicator in epidemiology to represent whether an infectious disease develops a pandemic.

$R_0$  is defined as the number of secondary infections that will result from a single infected individual entering a fully susceptible population

This flow scheme is described in figure, with the mathematical formulation as a system of three common differential equations:



Source: Thomas et al.(2020)

Transition from three possible states, S (t) sensitive, infected, I (t), and R (t) removed. The percentage of interactions between individuals in S (t) and I (t) leading to infection per unit time is r, and the percentage of infected individuals who recover with immunity or die from disease per unit time is  $\gamma$ . The total number of individuals within the system remains constant throughout the calculation.

Here, r decreases as the number of infected persons increases more, reflecting the increase in social distancing during the peak of infectivity. On the other hand, r increases when the number of infected persons decreases

The SIR model is simulated for different values of the constants  $\beta$  and  $\gamma$ . The population is normalized so that the simulations show percentage of the population in each category. So, for example, the number of infected is divided by the total number of the population to give the fraction of the population that is infected. The populations of the three strata are S (t), I (t) and R (t) at time t. Assume  $S (t) + I (t) + R (t) = N$  for all t.

Consequently,

so  $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$  people move from one category to another.

The model is described by the following equations

$$\frac{d(\frac{S}{N})}{dt} = -\beta si$$

$$\frac{d(\frac{I}{N})}{dt} = \beta si - \gamma i$$

$$\frac{d(\frac{R}{N})}{dt} = \gamma i$$

where  $\beta = \alpha \sigma \frac{S}{N}$  .represents the number of contacts per day that are sufficient to spread the disease.

Here  $\sigma \frac{S}{N}$  represents the number of healthy people the infected person meets during the day and infects  $\alpha \sigma \frac{S}{N}$ . and  $\gamma$  is the removal rate.

The complex models of Carletti (2020) require more parameters which are hard to distinguish, and we do not know for sure if they display a correct COVID -19 progression.

According to Rojas (2020), the Sir model can gather information, to have a clear image of the progression of the Pandemic, as it only has two parameters which are  $\beta$  and  $\gamma$ . These parameters are understood from the epidemiologists and can be calculated easily from the real data.

In the Sir models, the population of sensitivity, S, decreases monotonously towards zero. As a result, the level of infections rate tends towards zero, only if the population of sensitivity goes towards zero. The model explains the rapidly increase in the level of infection worldwide when infected people travel abroad and therefore this increases the level of infected people. The Sir Model, according to Cooper (2020), excludes most complications which are associated with the evolution in real time of the spread of the virus. An infected individual could stay infected and can be excluded from the population of the infected if he/she either recovers or dies. It is assumed that the time scale of the SIR model is quite short in order that natural births and deaths, disregarding the deaths from the virus, can be neglected and the overall number of deaths from the virus compared to the whole population remains quite low.

The model equations based on this assumptions and concepts, show the different intervals of change on the 3 layers of the population (Cooper & al, 2020). When there is a low number of infected individuals, the number of infections in the beginning tends to increase slowly looking like an exponential growth. What is being aimed often, is to flatten the curve as this will lower the peak of infections, and therefore the duration of the Pandemic is increased. On these conditions, to avoid high admissions in hospitals and to reduce fatalities as much as possible, restriction implementation is a necessity to keep in control the spread of the virus.

## NUMERICAL SIMULATION RESULTS

The simulations shown in the figures below show that when  $\beta$  increases (i.e. when the average number of contacts of an infected person that is sufficient to overcome the virus increases), the number of infected people reaches a higher number at the peak of the pandemic and the pandemic ends faster in time.

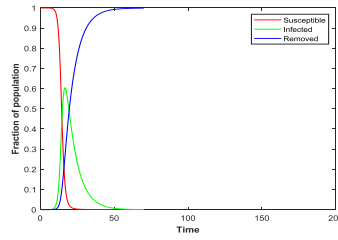


Figure 1:  $\beta = 1.1$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $MaxTime = 70$

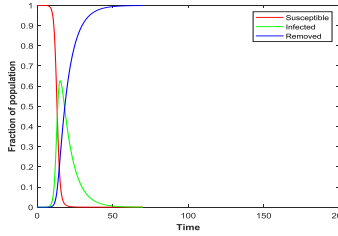


Figure 2:  $\beta = 1.2$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $MaxTime = 70$

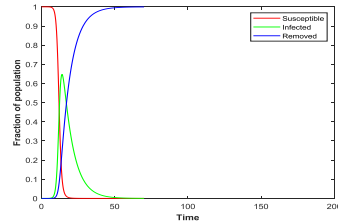


Figure 3:  $\beta = 1.3$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $MaxTime = 70$ .

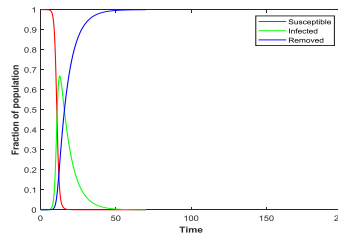


Figure 4:  $\beta = 1.4247$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $MaxTime = 70$

The figure above clearly shows that when Beta is 1.1, ie the average number of contacts of an infected person exceeds the number 1, the number of infected people reaches approximately 59 percent of the population at the peak of the pandemic and the pandemic ends quickly in time.

With the increase of beta from 1.1 to 1.2 and then to 1.4547, we notice that the number of infected reaches approximately from 59 percent of the population at the peak of the pandemic to 61 percent and when the beta reaches 1.4547 this percentage goes to approximately 67 percent, reflecting at the same time and shortening the time of the pandemic end, but in the conditions of a collapsing health system, this would be accompanied by an increase in fatalities.

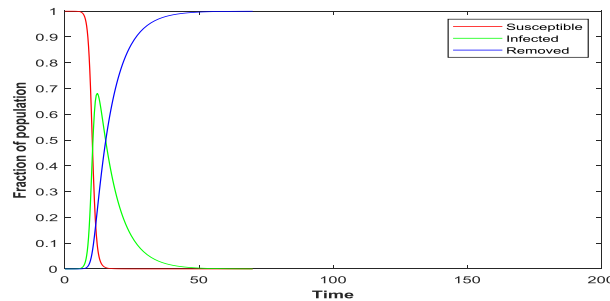


Figure 5:  $\beta = 1.5$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $\text{MaxTime} = 7$

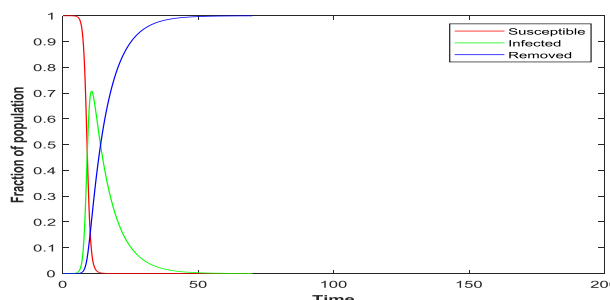


Figure 6:  $\beta = 1.7$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $\text{MaxTime} = 70$

The rate of spread is dictated by  $\beta$  which indicates the number of people a person must meet and spreads the virus. gradually the infected either recover or die and this is indicated by  $\gamma$  which is the removal rate. When the majority of the population is infected, the graph begins to decline and meanwhile the blue graph indicates an increase in the number of abortions.

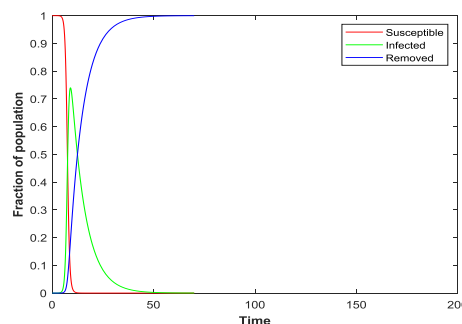


Figure 7:  $\beta = 2$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $\text{MaxTime} = 70$

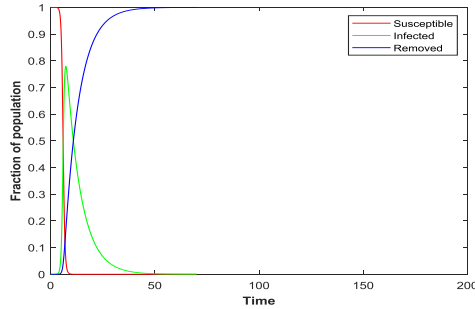


Figure 8:  $\beta = 2.5$ ;  $\gamma = 0.14286$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $MaxTime = 70$

When the beta still rises from 1.5 to 2.5, so the average number of contacts of an infected person quickly exceeds more than two, the number of infected reaches approximately 68 percent of the population at the peak of the pandemic to approximately 78 percent, the curve is still sharper and still shortens the time of pandemic termination.

Especially in the difficult conditions of overcrowding of hospital beds and follow-up of the sick, this dictates the necessity of severing the contacts of infected people with others.

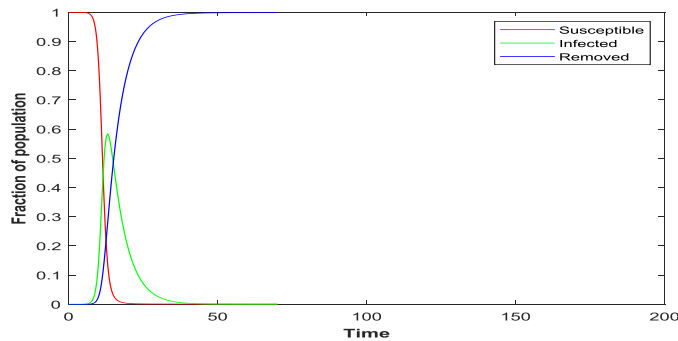


Figure 9:  $\beta = 1.4247$ ;  $\gamma = 0.2$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $MaxTime = 70$

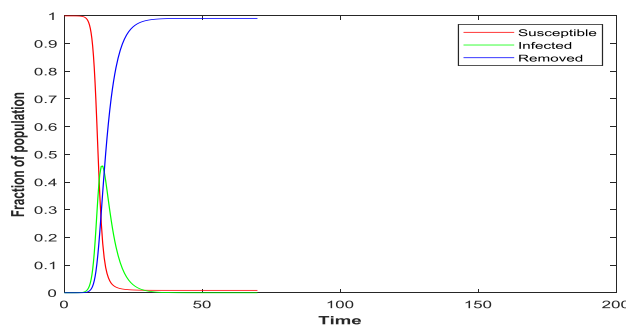
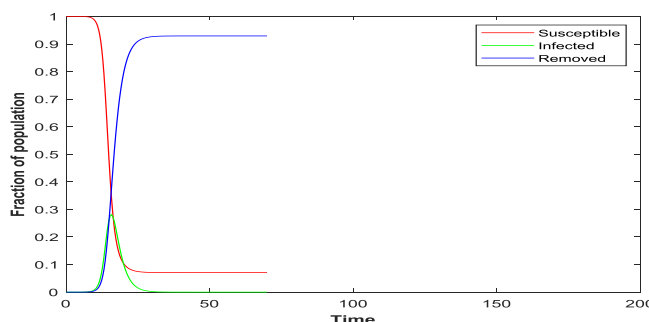
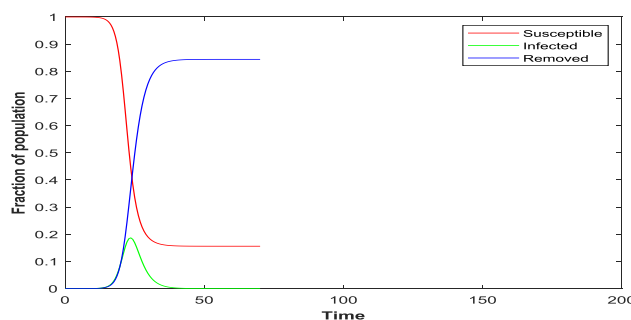


Figure 10:  $\beta = 1.4247$ ;  $\gamma = 0.3$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $MaxTime = 70$



Figure 2:  $\beta = 1.4247$ ;  $\gamma = 0.5$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $\text{MaxTime} = 70$ Figure 3:  $\beta = 1.1$ ;  $\gamma = 0.5$ ;  $S_0 = 1 - 1e-6$ ;  $I_0 = 1e-6$ ;  $\text{MaxTime} = 7$ 

The same behavior is observed when  $\gamma$  (percentage of infected people who recover or die within a unit of time) decreases; from the simulations it is noticed that the number of infected is more sensitive when it changes  $\gamma$  compared to the change of  $\beta$ . For  $\gamma = 0.5$ , the model shows that a part of the population (approximately 20%) never contracts the virus. The red graphic, when attempting near the ox axis, indicates that almost everyone is infected. When  $\beta$  passes the number 2 and the graph shows that approximately 80% of the population is infected in less than 1/6 of the time. Increasing the value of  $\gamma$  indicates that people recover faster. But if this is accompanied by death, he should take care of his health. Simulations show that social distancing is necessary to reduce infections.

## CONCLUSIONS

Given that for some reason systematic and complete data are missing in Albania, the SIR model, although it is the simplest model, describes the spread of coronavirus under the effect of social measures taken by governments and predicts the future trend of the spread of infection. The models assume that there is no second infection. By changing the constants of the model, such as the percentage of cases detected, the model manages to present different scenarios of the extent of the pandemic spread which is important for policymakers. Through

numerical simulations, this paper also shows that social measures taken by governments, such as distancing, vaccinating, using masks, are effective and lead to alleviating the pandemic.

Even though the SIR model has a lot of advantages, there are still some restrictions which have to do with the predicative ability as of the limited accurate information available. In the model, density and displacement of the population are not taken into consideration, supposing that there is a homogenous displacement of population. The model does not include parameters like humidity and temperature in the spread of the virus and reinfections are not considered, which until now have been excluded from the epidemiologists.

The SIR model is a good choice for the short-term period of this Epidemic, even though, there are restrictions in this model which are explained in the Marinova's Literature (2020), in case of a long-term infective disease.

The model can also be used to assess the reduction in the number of infected individuals, the reduction of the impact of the pandemic which requires that the distancing measures must be maintained for a period of time during the pandemic phases, in order to flatten the infection curve.

In this period when the achievement of herd immunity is required, and there is more data, epidemiological models may become more complex to make a realistic presentation of the situation. Studies using projection models to develop hypothetical scenarios are important in the future.

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