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TAIL DEPENDENCE IN THE AFRICAN STOCK MARKETS

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Abstract

This paper investigates the asymptotic (in)dependence of three major stock indices in Africa namely, the Nigerian Stock Exchange (NSE) All Share Index, Johannesburg Stock Exchange (JSE) All Share Index and Nairobi Stock Exchange 20 (NSE 20) Index using non-parametric tail dependence measures stemming from the bivariate Extreme Value theory - the chi and chi-bar measures. Likelihood ratio test for all pairs consistently permits us to reject the asymptotic dependence hypothesis. The NSE 20-NSE pair exhibits the highest left tail dependence while the highest for the right tail is the NSE 20-JSE pair. Within the class of asymptotic independence, positive association is the type of asymptotic independence exhibited by all pairs. These results make for more appropriate decision making by market participants especially when faced with different market situations.

Keywords: Bivariate returns, Extreme value theory, Financial markets, Tail dependence, Chi and Chi-bar measures

INTRODUCTION

One of the major factors that need to be considered when dealing with more than one stock market return is the structure of the dependence between the markets. The reason being that it affects portfolio selection, asset pricing and risk management. The need to have a good grasp of the dependence structure is an issue of utmost importance given the recent financial crisis that have rocked the financial world leading to unexpected huge losses and chain-like effect crashes.

The tail dependence concept is a technique that aids one in describing the dependence between extreme data, and the tail dependence coefficient which is one way of estimating the tail dependence (Sibuya, 1960) enables us to answer the question: "Will a crash in market A lead to a crash in market B?" Two forms of extremal dependence exist (asymptotic dependence and asymptotic independence) and dependence for fairly large values of each variable is permitted in both forms. Asymptotic dependence however, only occurs when very large values of each variable move simultaneously. A number of studies exist in literature that specifically assess the tail dependence structure of selected financial markets. The asymptotic dependence for three major stock markets within Europe was investigated by Poon et al. (2003). The approach used was the bivariate extreme value theory (EVT) dependence measure. Their results indicate that when the return series data is filtered for heteroscedasticity, the stock markets do not show statistically significant tail dependence. A similar study was undertaken by Fernandez (2003) where the EVT method was used to quantify value at risk (VaR) and the dependence in world markets. Considering 21 pairs of emerging stock market daily returns, Vaz De Melo Mendes (2005) investigated how their dependence behaves during a crisis period using extreme value copula functions. Still on the European markets, we have had Trzpiot and Majewska (2014) and Fortin and Kuzmics (2002) study the extreme dependence between different markets in that region. Singh et al. (2017) applied the nonparametric pair measures Chi (χ) and Chi-bar ($\bar{\chi}$) which are based on the bivariate EVT to investigate the asymptotic dependence of the Australian ASX All Ordinaries daily returns with 4 other international markets. Using the symmetrized Joe-Clayton copula, Bhatti and Nguyen (2012) studied the dependence structure across the Australia, US, UK, Hong Kong, Taiwan, and Japan financial markets. Based on the Chinese market, the tail dependence of an individual stock in relation to the whole market was analyzed by Fan et al. (2015). They further investigated the effect of this dependence on stock returns. Abberger (2005) made use of the chiplots to explore tail dependence in stock return pairs. Chen et al. (2010) also examined Chinese stock market and its extreme dependence with other major international stock markets.

The markets within the African continent are becoming very attractive investment options for global investors, therefore it is important for these investors to understand how the markets move especially in extreme situations such as in crisis periods. Empirical evidence of the dependence structure of stock markets in Africa are very few. Of note are studies by Mensah and Alagidede (2017), Ajasi and Biekpe (2006), Alagidede et al. (2011). Most of these studies however, do not focus on the tail dependence structure exhibited by the market extremes. In view of this, the objective of this paper is to evaluate the extreme dependence between three large markets in Africa namely, South African JSE All Share in the southern part of Africa, Kenyan NSE 20 in East Africa and Nigerian NSE All Share in West Africa. These markets are amongst the highly rated stock markets within their respective regions; hence their analysis serves as a good pointer to the nature of tail dependence between these regions. The χ and $\bar{\chi}$ plots are first used to explore the data for asymptotic dependence and then the coefficient of tail dependence and the likelihood tests are implemented.

The paper is designed as follows. Section 2 briefly explains what multivariate extremal dependence is and focuses on the measures of extreme dependence/independence that are used in the empirical analysis. Section 3 is devoted to the empirical exercise. The results are also discussed here. Conclusions are made in section 4.

MULTIVARIATE EXTREMES AND TAIL DEPENDENCE

Let $(X_i)_{1\le i\le n}$ be a sequence of n independent and identically distributed (iid) random vectors of d dimension from a given distribution function (df) F. The limiting behaviour of the component-wise maxima $(max_{1\le i\le n}X_{i,1}, \dots, max_{1\le i\le n}X_{i,d})$ forms the foundation upon which multivariate EVT is built. Assuming that for every positive integer n, vectors $b_n > 0$ and a_n exist and a df G having non-degenerate margins such that

$$P\left\{max_{1\le i\le n}\frac{X_i - b_n}{a_n} \le x\right\} = F^n(a_n + b_n) \to G$$

as $n \to \infty$, then we call G a multivariate extreme value distribution and F is said to be in the (multivariate) domain of attraction of G. For this study we focus on bivariate data.

The χ and $\bar{\chi}$ dependence measures

Based on the concept of tail copulas, these measures were developed by Heffernan (2000), Coles et al. (2000), and Poon et al. (2003, 2004) to quantify the asymptotic (in)dependence between two sets of random variables X and Y. There are basically four types of dependence relations in a multivariate setting: independence, perfect dependence, asymptotic independence (extreme values of X and Y occur at different times), and asymptotic dependence (extreme values of X and Y occur simultaneously). All of these possibilities are covered under this method.

χ and $\bar{\chi}$ are two nonparametric measures of tail dependence we use to describe the levels of asymptotic dependence and asymptotic independence. The main assumption here is that the data is sampled independently. We consider two series S and T which are transformations of the bivariate returns (X, Y) in terms of their cumulative distribution functions (cdf). Then clearly, the events $\{S > u\}$ and $\{T > u\}$ as the threshold $u \to 1$ correspond to equally extreme events for each variable. χ, which is the measure of dependence, is defined as

$$\chi(u) = \lim_{u\to\infty} \mathrm{P}(\mathrm{T} > u, S > u) = \lim_{u\to\infty} \frac{\mathrm{P}(\mathrm{T} > u, S > u)}{\mathrm{P}(\mathrm{S} > u)} \qquad 0 \leq \chi \leq 1$$

We note that,

$$\mathrm{P}(\mathrm{T} > u | S > u) = \frac{\mathrm{P}(\mathrm{T} > u, S > u)}{\mathrm{P}(\mathrm{S} > u)}$$

$$= \frac{\mathrm{P}(\mathrm{T} < u, S < u) + \mathrm{P}(\mathrm{T} > u) + P(S > u) - 1}{1 - \mathrm{P}(\mathrm{S} < u)} \qquad (1)$$

$$= \frac{1 - 2\mathrm{u} + C(u,u)}{1 - \mathrm{u}} = \frac{2(1 - \mathrm{u}) - 1 + C(u,u)}{1 - \mathrm{u}}$$

$$= 2 - \frac{1 - C(u,u)}{1 - \mathrm{u}} \sim \frac{2 - \log C(u,u)}{\log u} \quad as\ u \to 1$$

Thus in terms of copulas, $\chi(u)$ can also be

$$\chi(u) = \frac{2 - \log\ \mathrm{P}(\mathrm{U} < u, V < v)}{\log \mathrm{P}(\mathrm{U} < u)} \quad 0 \leq u \leq 1$$

It can easily be shown that $\chi = \lim_{u\to 1} \chi(u)$

$\chi > 0 \Rightarrow S\ and\ T$ are asymptotically dependent; $\chi > 1 \Rightarrow S\ and\ T$ are perfectly asymptotically dependent; $\chi = 0 \Rightarrow S\ and\ T$ are asymptotically independent.

The function $\chi(u)$ is a quantile dependent measure of dependence. The sign of $\chi(u)$ determines whether the variables are positively or negatively associated at the quantile level u. Given that two series may be asymptotically independent (i.e. no co-movements in their large values) but may show some degree of dependence for finite levels of u which is significant, we therefore, make use of a complementary measure $\bar{\chi}$. $\bar{\chi}$ is the measure of the degree of finite dependence when the variables are asymptotically independent. It is defined by Coles et al. (1999) as

$$\bar{\chi} = \lim_{u\to 1} \frac{2\log\ P(S > u)}{\log P(T \geq\ u, S \geq u)} - 1 \qquad -1 \leq\ \bar{\chi} \leq 1$$

$$= \lim_{u\to 1} \frac{2\log(1-u)}{\log C(u,u)} - 1 \qquad (2)$$

$\bar{\chi} > 0 \Rightarrow$ positive dependence between the finite series; $\bar{\chi} < 0 \Rightarrow$ negative dependence between the finite series; $\bar{\chi} = 0 \Rightarrow$ exact independence between the finite series; $\bar{\chi} = 1 \Rightarrow$ perfect dependence between the finite series.

We can then use the pair $(\chi, \bar{\chi})$ to provide all the necessary information relating to the degree and type of extremal dependence for any two series. For asymptotic dependent variables $\bar{\chi} = 1$ and $0 \leq\ \chi \leq 1$. Here χ is used to quantify the degree of dependence. For the case of asymptotic independence, $\chi = 0$ and $-1 \leq\ \bar{\chi} \leq 1$. $\bar{\chi}$ is used to measure the degree of dependence. Thus, in practice it is important to first check if $\bar{\chi} = 1$.

Estimating χ and $\bar{\chi}$

The hill estimator

The bivariate returns (X, Y) having marginal cdf F_x and F_y respectively, are first transformed into standard Fréchet marginals (S, T) in the following way:

$$S = -\frac{1}{\log F_X(\mathrm{X})} \qquad T = -\frac{1}{\log F_Y(\mathrm{Y})} \qquad S > 0, T > 0$$

Now the variables (S, T) have the same dependence structure as (X, Y). The tail of a Fréchet-type univariate variable above a threshold u follows the model

$$P(Z\ >\ z) \sim L(z) z^{-\frac{1}{\xi}} \qquad z > u$$

with ξ being the shape parameter or the tail index and $L(z)$ a slowly varying function of z. That is,

$$S = \lim_{u\to\infty} \frac{L(uz)}{L(u)} = 1 \quad z > o$$

Assuming that the data are iid and $L(z)$ is a constant d, then the maximum likelihood estimate $z_{(1)}$ (mle) of ξ (which is the Hill estimator) and d are

$$\xi = \frac{1}{n_u}\sum_{j=1}^{n_u}\log\frac{z_{(j)}}{u} \quad and \quad d = \frac{n_u}{n}u^{1/\xi}$$

where $z_{(1)}, \dots, z_{(n_u)}$ are the n_u observations of Z above the threshold u.

Therefore, since S and T are Fréchet-type variables, Ledford and Tawn (1996, 1998) applied the same procedure.

Assumptions

The four assumptions required to estimate χ and $\bar{\chi}$ are:

1. The joint tail behaviour of the distribution (S,T) is bivariate regularly varying, satisfying Ledford et al. (1998) conditions.
2. The limiting behaviour is reflected by the sample characteristics of the empirical joint distribution above a specified threshold.
3. Sufficient independence over time is required by the series in order for the sample characteristics to converge to the population characteristics χ and $\bar{\chi}$.
4. One can transform the marginal variables to identically distributed Fréchet variables. Ledford and Tawn (1996, 1998) showed that under weak conditions

$$P(S > u, T > u) \sim L(u)u^{\frac{1}{\eta}} \quad as \quad n \to \infty$$

η is the coefficient of tail dependence and it lies in the interval $(0, 1]$, $L(s)$ is a slowly varying function.

Estimating η

In order to estimate η, they made use of the structure univariate variable $R = min(S,T)$. Drawing from the fact that from univariate EVT $P(R > u) = P(min(S,T) > u) = P(S > u, T > u) \sim L(u)u^{-1/\eta}$, it implies that when the conditional probability is considered, we will have for large u,

$$P(R > u + t \mid R > u) = \frac{P(R > u + t, R > u)}{P(R > u)}$$

$$\sim \frac{L(u+t)(u+t)^{-1/\eta}}{L(u)u^{-1/\eta}} \qquad (3)$$

$$= \frac{L(u+t)(u+t)^{-1/\eta}}{L(u)u^{-1/\eta}} \left(\frac{u+t}{u}\right)^{-1/\eta}$$

$$\sim \left(1+\frac{t}{u}\right)^{-1/\eta}$$

Comparing this result with the generalized Pareto distribution (GPD)

$$H(t) = 1 - \left(1+\xi\frac{t}{\sigma}\right)_{+}^{-\frac{1}{\xi}} \sim \left(1+\xi\frac{t}{\sigma}\right)_{+}^{-\frac{1}{\xi}}$$

we have $\xi = \eta$ and $\sigma = \eta u$. This means that the excesses above u when Z is considered follows a GPD with shape parameter η and scale parameter ηu . It can be shown that $\chi = 2\eta - 1$ and $\chi = \frac{n_u}{n} u$ where η is estimated using the hill estimator. An increasing value of η indicates a stronger association and given a specific η, the relative strength of dependence is given by L. For a more detailed explanation of the estimation method see Poon et al. (2003, 2004). Heffernan (2000) identified three types of asymptotic independence within the class of asymptotic independent variables. They are, positive association ($1/2 < \eta < 1$ or $\eta = 1$ and $L(u) \to 0$ as $u \to 1$), negative association ($0 < \eta < 1/2$) and the near independent case ($\eta = 1/2$ but when $L(u) = 1$, perfect independence is attained).

Log-likelihood ratio (LLR) test for asymptotic (in)dependence

The LLR test statistic given as $D = 2(logL_1 - logL_0)$ has approximately, a chi-square distribution with one degree of freedom. The test for asymptotic dependence has the null hypothesis as H_0: $\eta = 1$.

EMPIRICAL ANALYSIS

Data from the years 2005 to 2015 is used and it consists of daily return data series having 2700 log returns each. A main advantage of this time period is the inclusion of the 2007 financial crises. This provides us with very large extremes which are a core requirement in tail dependence analysis. The data has not been filtered for heteroskedasticity. The log plots (Figure 1) for each pair of stock markets indicates the presence of dependency because higher values of one market are associated with higher values of the other market. However, we need to test further to confirm if tail dependence exists.

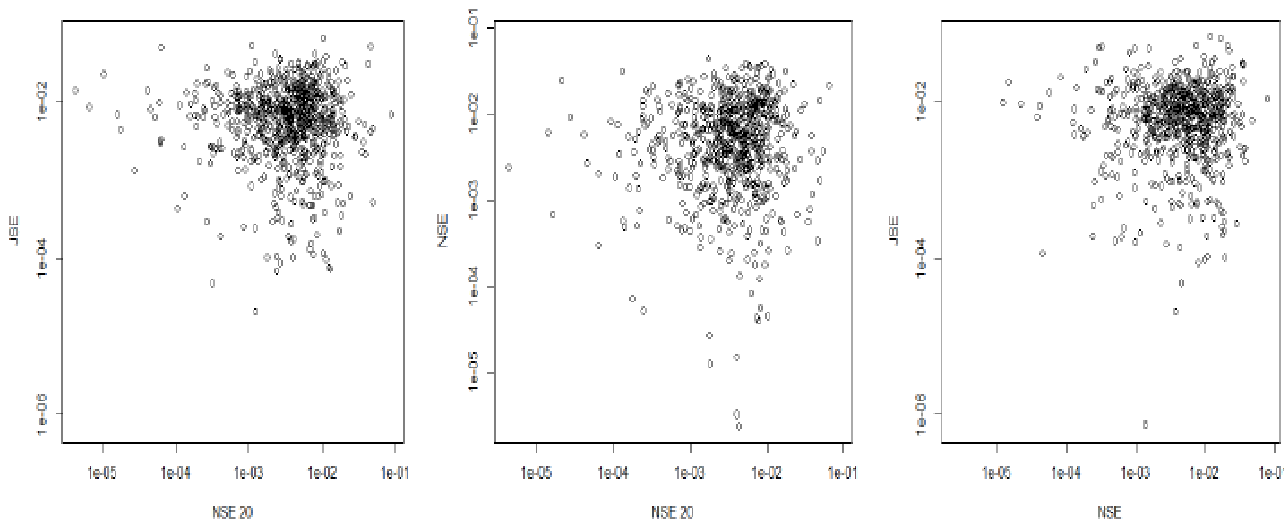


Figure 1: Log plot for the 3 markets pairs.
NSE 20-JSE (left), NSE 20-NSE (middle) and NSE-JSE (right).

A general observation for the right tails (Figure 2) shows that for lower values of u the χ plot is greater than zero but as u increases, χ reduces for all pairs. The point-wise confidence intervals cover a wider range of possible limits. Specifically, in the case of the NSE 20-JSE pair, for u greater than 0.9, χ is greater than 0 but its confidence interval includes 0 and 1. On the other hand, the $\bar{\chi}$ measure for same pair is less than one. These results do not give us a clear picture of the nature of the extremal dependence between these markets. For the NSE 20-NSE and NSE-JSE pairs, χ is approximately zero for large values of u and $\bar{\chi}$ is less than one. This indicates asymptotic independence for both pairs. The left tails showcase a slight upward trend for χ at higher values of u. This is in contrast to the right tail observations. χ is a bit higher than zero for the NSE 20-JSE and NSE 20-NSE markets but zero for the NSE-JSE set. Again $\bar{\chi}$ is less than one in all cases. The results for the latter indicate asymptotic independence but for the former two sets, it is difficult to decide on the type of tail dependence exhibited by the pairs. To overcome these sort of issues, more formal tests are incorporated. We employ the univariate EVT to estimate η, taking advantage of its relationship with $R = min\,(S,T)$ (see section 2.2). To assist with threshold choice, the plots in Figure 3 are used. A rough linear line can be observed between points 0.8 and 0.9 for u in diagram 'a' thus we choose the 80th percentile of R as the threshold.

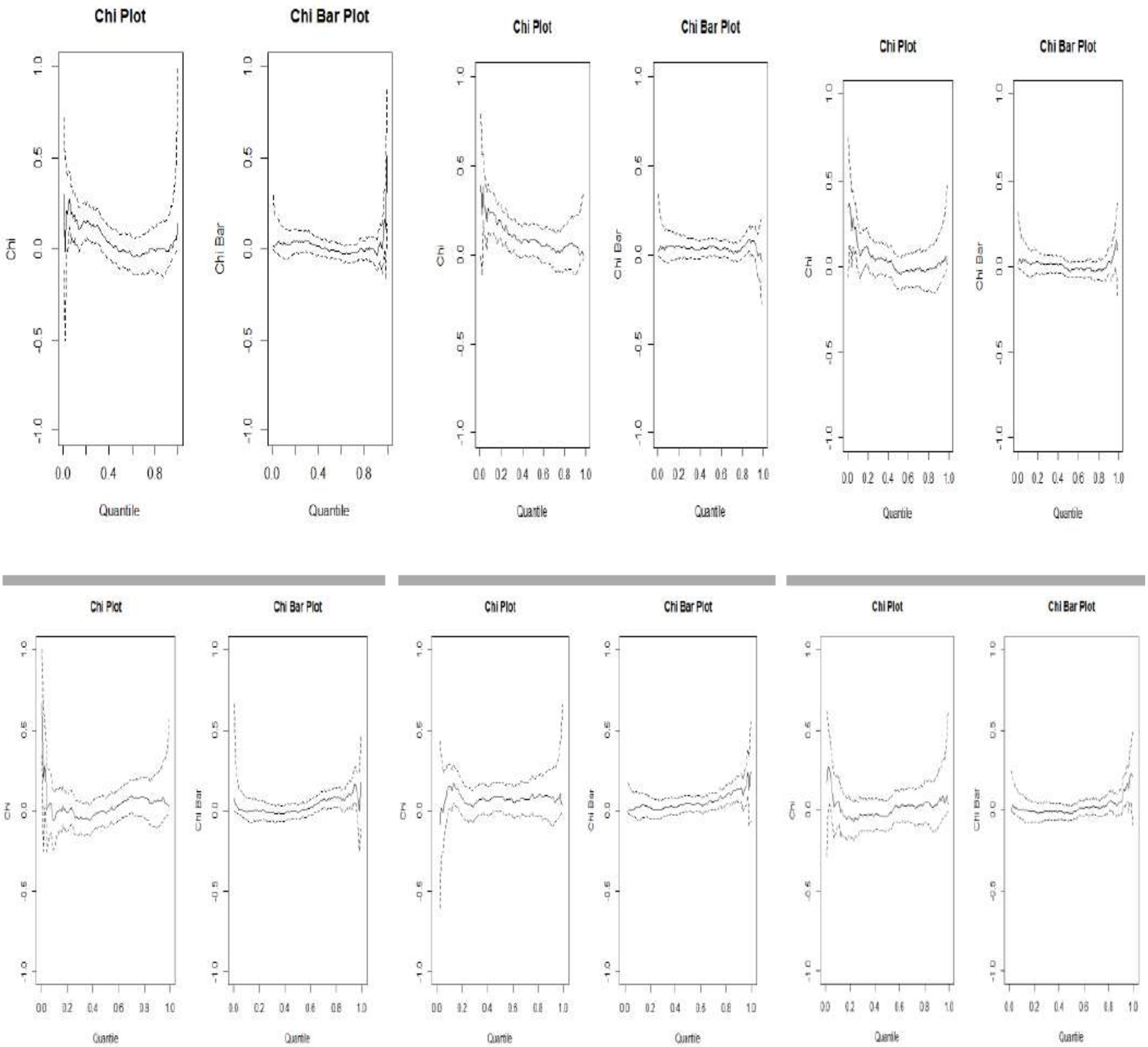
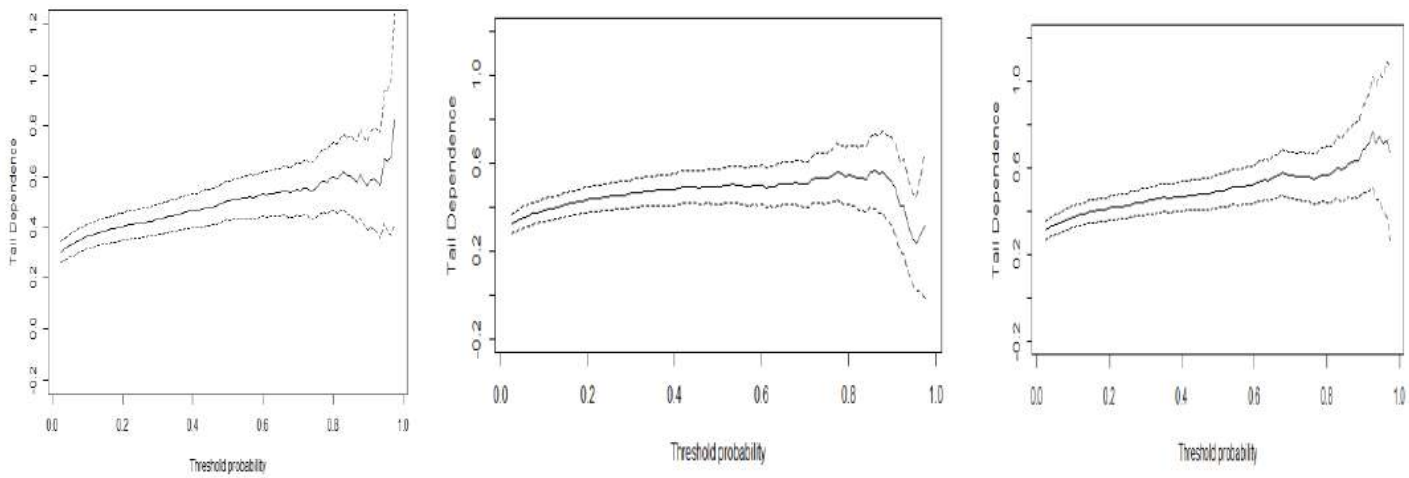


Figure 2: χ and $\bar{\chi}$ plots for the market pairs. Right tails (above) and left tails (below) of the pairs NSE 20-JSE (left) NSE 20-NSE (middle) and NSE-JSE (right)

Similarly, (going from left to right) for both tails, approximate stable regions occur between points 0.7 and 0.9; 0.7 and 0.8; 0.5 and 0.7; 0.7 and 0.9; 0.6 and 0.8. The following percentiles are therefore chosen as threshold: the 75th, 75th, 50th, 70th and 60th. The left tail of the NSE 20-NSE markets has the highest value of η followed by the NSE-JSE return pair (Table 1). This means that there exists a stronger association between the NSE 20 and NSE markets than that of the NSE and JSE markets. Overall, η is not equal to 1 implying that all stock pairs exhibit asymptotic independence in both tails. This is further confirmed by the LLR test for asymptotic dependence allowing us to reject the null hypothesis at the 5% significance level.

In bear markets (left tails), the tail dependence between the Kenya-Nigeria and the Nigeria- South Africa returns are higher while the reverse is the case for the Kenya-SA market. Since all market sets have $0 < \eta < ½$, this suggests that within the class of asymptotic independence they exhibit the positive association-type of asymptotic independence.



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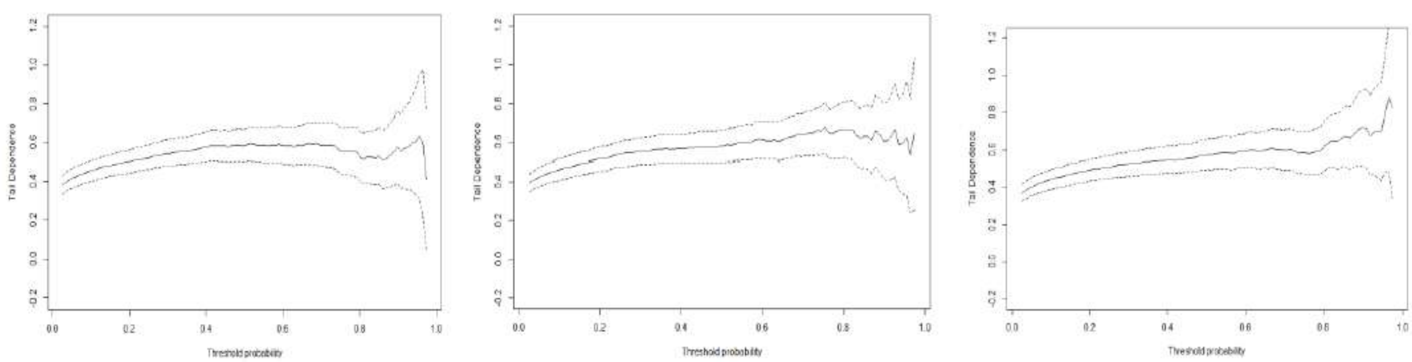

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Figure 3: Tail dependence coefficient plots for different thresholds. Right tails (above) and left tails (below) of the pairs NSE 20-JSE (left) NSE 20-NSE (middle) and NSE-JSE (right)

This implies that more observations occur for which both X and Y exceed the threshold u than in the exact independent case.

CONCLUSION

This paper employed exploratory and more formal diagnostics to investigate the extremal dependence structure between selected markets in Africa. The left and right tails of the unfiltered returns were analyzed. Asymptotic independence was observed for all pairs of stock returns. The left tails (bear markets) of the stock returns between the Kenya and Nigeria markets display the strongest tail dependence and for the bull markets (right tails), the Kenyan and South African stocks are more dependent. Overall, this analysis provides useful insights on how markets, within the African region, relate. This is valuable information to risk managers and market participants. A possible extension of this study is to incorporate portfolio management based on the results obtained. The effect of heteroskedasticity on the results could also be examined.

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