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A SIMPLIFIED REVIEW OF ROMER'S (1990) MODEL

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Abstract

This paper analyzes Romer's(1990) model without any argue to Romer's growth model and conundrum surrounds the growth matter. The model filled gap in the literature and enhances economists' considerate an endogenous technological change. However, given its complexity and difficult to demonstrate how the level of R&D is determined in the general market equilibrium within the Romer model. This paper provides a simplified way to understand the Romer model by presenting both consumption and production sides of the economy, the latter is made up of three productive sectors: the final goods, producer durables and R&D sectors. The paper also derives step by step all sectors but not too far ahead in entire advanced mathematical calculus and algebra for the intuition purpose. Thus, this layout sheds light on the Romer model and become more accessible to enhance the theory of the knowledge- based economy.

Keywords: Economic Growth, Endogenous Growth, Romer model, Human capital

INTRODUCTION

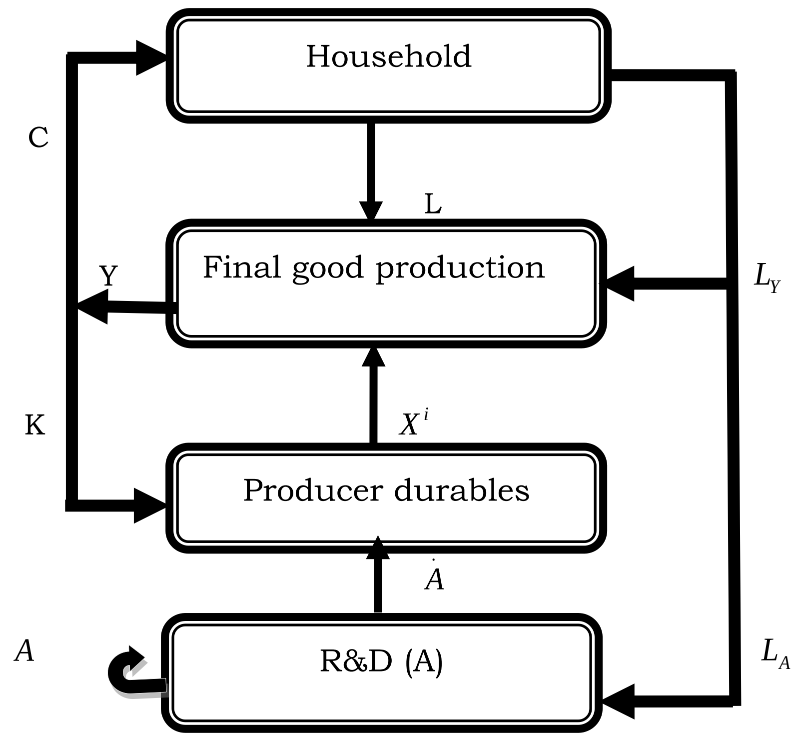
Today, the Romer (1990) model is central to study the economic growth. The model filled gap in the literature and enhances economists' considerate an endogenous technological change. However, given its complexity and difficult to demonstrate how the level of R&D is determined in the general market equilibrium within the Romer model.

This paper provides simplified way to understand the Romer model by presenting both consumer´s and production sides of the economy, the letter is made up of three productive sectors: final goods, producer durables and R&D sectors. We derive step-by-step all sectors but not too far ahead in entire advanced mathematical calculus and algebra for the intuition purpose. Our analysis is based on the form presented by Jones (1995) and Aghion and Howitt (1998) without any argue to Romer's growth model and conundrum surrounds the growth matter.

This paper is structured into three sections after the introduction; anatomy of the model is presented in section two while the conclusion comes last.

ANATOMY OF THE MODEL

The following is simplified skeleton of standard Romer's1990 model



Source: Romer(1990)

The consumption side

Let depict the household sector represented by one infinitely living individual who maximizes the discounted stream of utilities over an infinite time horizon subject to his budget constraint. Formally

$$Max\int_0^{\infty} e^{-\rho t}U(C_t)dt \text{, with } U(C_t)=\frac{C^{1-\sigma}-1}{1-\sigma} \quad \text{………………………………………………………..…… (1)}$$

Where variable C_t is aggregate consumption in period t, ρ is the rate of time preference, and $\frac{1}{\sigma}$ is the elasticity of substitution between consumption at two periods of time. The representative consumer facing a constant interest rate r, opt to have consumption growing at the constant rate gc given by the Euler equation:

$$gc = \frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho) \quad \text{.........................(2)}$$

The production side

The model is structured by three sectors: final good sector, intermediate goods sector and R&D sector. The final output sector Y produces output that can be used for consumption using labor L_Y and intermediate goods, i, that are available in A varieties each produced in quantity $x(i)$,and the production function is given by:

$$Y_t = L_Y^{1-\alpha} \int_0^{A_t} x_t(i)^{\alpha} di \quad \text{.........................(3)}$$

For A constant, the production function with constant return to scale in L_Y and $x(i)$, and diminishing return in $x(i)$ for L_Y fixed. Hence, the technological growth that us continuous increases in A, avoid the tendency for the diminishing returns to rise in $x(i)$.The capital accumulation equation is then: $\dot{K}_t = Y_t - C_t$

Noting that it takes the one unit of foregone consumption to produce one unit any type of capital good, K is related to the capital good refer to the subsequent rule:

$K_t = \int_0^{A_t} x_t(i) di$, route of accumulation of new designs, the production function of new design is:

$$\dot{A}_t = \delta L_{At} A_t \quad \text{.........................(4)}$$

Where L_A the total labour is employed in the research, the variable L_A and L_Y are linked by the constant by the constraint: $L_{At} + L_{Yt} = L_t$, anyone can be assign in either to the final goods or to the research. The specification in (4), all researchers have an access to the total stock of knowledge A. In this model, knowledge appears in production function in two divergent aspects: First, a new design matches to new capital goods which utilized to produce final good. Also, a new design raises the total stock of knowledge and therefore raises the productivity of labour in the research sector. The owner of a design has property rights over the production of the respective capital good but not over use of the created design in the research sector.

Assuming first that allocating more labour to R&D leads to a higher growth rate A, second, the higher the total stock of A the higher the marginal productivity of the researcher. Third, the output of designs is linear in A, allowing the balanced growth path i.e equilibrium with a constant growth rate for A, K, Y and C.

In perfect competition setting, final good producers rent each capital good on to the profit maximization rule:

$\frac{dY_t}{dx_t(i)} = R_t(i)$,where, $R(i)$ is the rental price of each capital goods. This is the inverse demand function curve by each capital good producer:

$$R_t(i) = \alpha L_{Yt}^{1-\alpha} x_t(i)^{\alpha-1} \quad \text{……………………………………………………………………………...…..……..(5)}$$

With given values of r and L_Y, each capital good producer, who has patent already incurred the fixed cost investment in a design, P_A, and has the on it, will maximize its revenue minus variable cost at every date: $\max \pi_t(i) = R_t(i)x_t(i) - r_t x_t(i)$

The constant marginal cost and a constant elasticity demand curve, this monopolistic competitor solves his problem by charging a monopoly price which is a markup over marginal cost. The markup is determined by the elasticity of demand (α − 1).

$$\max \pi(i) = \alpha L_Y^{1-\alpha} x(i)^{\alpha} - rx(i), \quad \frac{d\pi(i)}{dx(i)} = \alpha^2 L_Y^{1-\alpha} x(i)^{\alpha-1} - r = 0$$

$$R(i) = \frac{r}{\alpha}$$

The intuition is that firm incurs a fixed cost when it produces a new capital and it recovers cost by selling its good for a price $R(i)$ that is higher than its constant marginal cost. The choice to produce a new capital good depends on the assessment between the discounted stream of net revenues that the patent on this good will bring in the future, and the cost P_A of the initial investment in a design. The R&D costs must be paid up front, prior to profits be earned. This time structure initiates natural dynamics in the model.

The market for designs is competitive, so at every date t the price for designs will equal to the present value of the future revenues that a monopolist can take out i.e capital goods producers earn zero profits in a present value sense. The dynamic zero-profit/free-entry condition is then:

$$P_{At} = \int_{t}^{+\infty} e^{-r(\tau - t)} \pi_{\tau}(i)\, d\tau \quad \text{………………………………………………………………………….……..(6)}$$

$\dot{P}_{At} = r_t P_{At} - \pi_t(i)$, assuming that there are no bubbles, the equation (5) can be presented as:

$$r_t P_{At} = \pi_t(i) + \dot{P}_{At}$$

This means that firms opt putting the monetary value P_{At} in the bank and earn interest on deposit, $r_t P_{At}$, or purchasing a patent for the equal value and earn the returns of producing the differentiated good, $\pi_t(i)$ plus the capital gain/loss of owning that patent, $\dot{P}_{At}$, means the Fisher equation of this model.

The model is solved for its balanced growth path, the equilibrium for which variables A, K, C and Y grow at constant exponential rates. The Euler equation (2), in a balanced growth path, the interest rate has to be constant thus $R(i)$, noting symmetric in the model, all producers have the identical technological features and face the same market condition therefore will pick the same equilibrium.

This implies that $R(i) = \bar{R} = R$ and $x(i) = \bar{x} = x$. Then, we rewrite the expressions for R_t and x_t: $R_t = \alpha L_Y^{1-\alpha} x_t^{\alpha-1}$ and, equivalently: $x_t = L_{Yt}\left[\frac{\alpha^2}{r}\right]^{\frac{1}{1-\alpha}}$ from which we can detect that in a balanced growth path, with L_Y constant (required for BGP, as explained below), x is also constant. Since all capital goods producers produce in the same quantity, total physical capital comes to:

$K_t = \int_{0}^{At} x_t(i)\,di = A_t x_t$ and the production function can be rewritten as:

$Y_t = L_{Yt}^{1-\alpha} A_t x_t^{\alpha}$, with L_Y and x constant, it is clear from log-differentiation of the two equations above that K and Y grow at the same rate as A. Now, rewriting the production function so that K appears specifically, we have:

$Y = L_Y^{1-\alpha} A x^{\alpha} \Leftrightarrow Y = L_Y^{1-\alpha} (Ax)^{\alpha} A^{1-\alpha} \Leftrightarrow Y = K^{\alpha} (L_Y A)^{1-\alpha}$, this is similar to Solow's production function.

The marginal productivity of capital is: $\frac{dY}{dK} = \frac{\alpha L_Y^{1-\alpha} A^{1-\alpha}}{K^{1-\alpha}}$

We observe that, for L_Y constant, with physical capital K growing at the same rate as technology A, the marginal productivity of capital is held constant. Thus, model provides sustained per-capita growth, in the manner that predicted by Solow's model. The technological progress defeats diminishing returns to capital and core of sustained positive growth in Romer's model has thus been identified. While in Solow's model, the growth rate of A is exogenous to the model, in Romer's model, this growth rate is determined within the model.
We explore how this growth rate is endogenously determined: the engine of growth is given by equation (4), recurred:

$\dot{A}_t = \delta L_{At} A_t$, imply that: $gA = \delta L_{At}$,i.e technological progress, gA, depends on L_A, the number of people that choose to work in the research sector. Equation (4) makes it clear that a balanced growth path solution, with a constant growth rate, requires that L_A remains constant. Thus, the existence of a balanced growth equilibrium requires that prices and wages are such that L_Y and L_A remain constant as A, K, Y and C grow at a constant exponential rate.

The distribution of workers among the final output and research sectors pursue the labour market equilibrium condition that remuneration of labour must be the same in both sectors. In the final goods sector, the wage paid to L_Y is:

$w_{Yt} = \frac{dY_t}{dY_{Yt}} = (1-\alpha) L_{Yt}^{-\alpha} A_t x_t^{\alpha}$, the research sector, reward is:

$w_{Yt} = \frac{d\dot{A}_t}{dL_{At}} P_{At} = \delta A_t P_{At}$, equality of the two implies that:

$$P_{At} = \frac{(1-\alpha)}{\delta} L_{Yt}^{-\alpha} x_t^{\alpha} \quad \text{……………………………………………………………………….(7)}$$

Log-differentiation of equation (7) shows that in a balanced growth path, as L_Y and x are both constant, P_A is also constant. Hence, with $\dot{P}_A = 0$, the zero-profit condition (5) becomes:
$0 = rPA - \pi$

$$r = \frac{\pi}{PA} \quad \text{……………………………………………………………………………………………….(8)}$$

Noting equation (4), and the markup $R(i)=\frac{r}{\alpha}$, we can rewrite the profits expression as:

$$\pi = Rx - rx$$

$$\pi = (1-\alpha)\alpha L_Y^{1-\alpha} x^{\alpha}$$ ………………………………………………………………………………….(9)

Substituting expressions (9) and (7) in to(8), we obtain

$$r=\frac{\pi}{P_A}=\frac{(1-\alpha)\alpha L_Y^{1-\alpha}x^{\alpha}}{\frac{1-\alpha}{\delta}L_Y^{-\alpha}x^{\alpha}} \Leftrightarrow r=\delta\alpha L_Y$$

, which is correspondent to:

$$L_Y=\frac{r}{\delta\alpha}$$ ……………………………………………………………………………………….(10)

Then, it follows that the growth rate of A is:

$$gA = L_A \Leftrightarrow gA=\delta\left(\bar{L}-L_Y\right) \Leftrightarrow gA=\delta\bar{L}-\frac{r}{\alpha}$$ …………………………………………………..(11)

As mentioned, output and physical capital grow at the same rate as A. And, as shown below, the capital accumulation equation implies that consumption also grows at the similar rate as Y and K. That is:

$\dot{K}=Y-C \Rightarrow \frac{\dot{K}}{K}=\frac{Y}{K}-\frac{C}{K}$ A constant gK implies that $\frac{d\left(\frac{\dot{K}}{K}\right)}{dt}=0 \Rightarrow \left(\frac{\dot{Y}}{K}\right)=\left(\frac{\dot{C}}{K}\right)$ which, due to $gY=gK$ implies that: $\left(\frac{\dot{C}}{C}\right)=\left(\frac{\dot{C}}{K}\right)$, with a constant population, this growth rate is the same as the per-capita growth rate: $g_c=g_y=gA=g$ and it is:

$$g=\delta\bar{L}-\frac{r}{\alpha}$$ ……………………………………………………………………………………….(12)

Finally, to solve this decentralized economy's problem, we determine the general equilibrium solution. In (12) shows pairs (g,r) of BGP on the production side. This is well known as Technology curve (Rivera-Batiz and Romer, 1991), showing a negative relationship between the interest rate and the growth rate.

The Euler equation (2) represents pairs (g,r) of balanced growth paths on the consumers side, showing positive relationship between the interest rate, r, and the growth rate, g,known as the Preferences curve (Rivera-Batiz and Romer ,1991).

The general equilibrium balanced growth path for this economy is determined where the two curves intersect as indicated in figure 1:

Rivera-Batiz and Romer (1991) spot out that a parameter restriction is crucial for the growth rate not to be greater than the interest rate. If not, present values would not be finite and restriction always met if $\sigma \geq 1$, i.e the Preferences curve lies on or above the 45^{o} line. If $\sigma \text{ p } 1$, then the Technology curve cannot lie too far up and to the right. The equilibrium growth rate is the solution to the system of two equations:

(11) and (2), and two unknowns: r , g :

$$g = \delta \bar{L} - \frac{r}{\alpha}$$

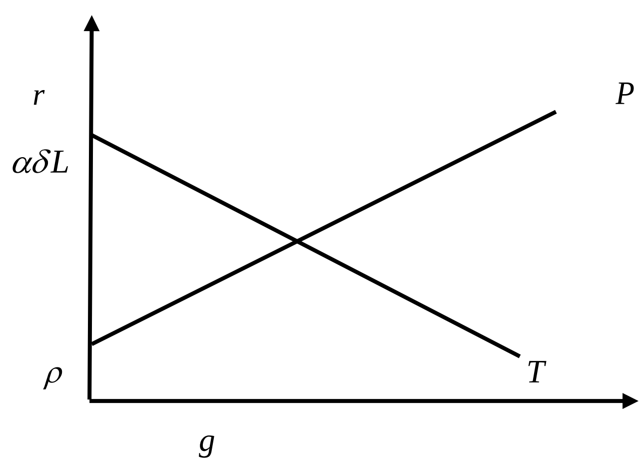
$$g = \frac{1}{\sigma}\left(r - \rho\right)$$

, means:

$$g = \frac{\alpha \delta \bar{L} - \rho}{\alpha + \sigma}$$ ……..(13)

Arnold (2000) provides a complete sketch of the dynamics of Romer's (1990)model in the neighborhood of its steady-state, showing the equilibrium of the model analyzed in terms of a system of three differential equations of three variables $x = \frac{C}{K}$, $Z = \frac{Y}{K}$ and L_Y. The steady-state of this system corresponds to the balanced growth path of Romer's model. He argues that there is a unique and monotonic growth path converging to the steady-state, which is a saddle point.

Figure 1: General equilibrium balanced growth path



The model does not have local indeterminacy, instability nor cycles, Arnold also exhibits that the initial value of $\frac{A}{K}$ solely determines the starting point on the saddle-path of the system. Equation (13) proves that, as opposed to the neoclassical model, in Romer's model equilibrium growth rate is influenced by the preference parameters σ and ρ. In a figure 1, if either of these two parameters falls, the Preferences curve shifts to the left, leading to a new equilibrium with a higher growth rate.

The equilibrium growth rate positively depends on the technology parameter α, the capital's share in total income. Moreover, economic growth is relatively to the size of the labour force, L (total population). In figure 1, a rise in L shifts the Technology curve to the right, leading to a new balanced growth path with a higher growth rate and a higher interest rate, this so called the scale-effects .The origin of this scale-effects insight in R&D equation (4), implies that technological growth is proportional to the level of labour allocated to research, L_A i.e economic growth is proportional to the size of the economy's population. The effect of integration of two identical economies is easily to show in (13): Integration doubles the size of the economy, into $2L$, rising the equilibrium growth rate and interest rate.

The equilibrium growth rate spoken in (13) is not optimal with two causes of non-optimality: The first, the capital goods producers charge a price that is exceeding the marginal cost. Remember the markup rule:

$R = \frac{r}{\alpha}$, noting also expression (5), $R = \alpha L_Y^{1-\alpha} x^{\alpha-1}$.

The marginal productivity of capital is: $\frac{dY}{dK} = \frac{\alpha L_Y^{1-\alpha} A^{1-\alpha}}{K^{1-\alpha}} = \frac{\alpha L_Y^{1-\alpha} A^{1-\alpha}}{(Ax)^{1-\alpha}} \Leftrightarrow \frac{dY}{dK} = \alpha L_Y^{1-\alpha} x^{1-\alpha}$

Hence, it pursues: $r = \alpha R = \alpha \frac{dY}{dK}$,i.e ,capital is rewarded under its marginal productivity.

The second, the presence of the externality created by the individual decision to do R&D not take into account the gains other R&D activities will get from.

The solution to the Social Planner's setup of model maximizes the representative consumer's utility:

$$\max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1-\sigma} dt \qquad \text{.....(14)}$$

Subject to the following constraints:

$$Y_t = K_t^{\alpha} L_{Yt}^{1-\alpha} A_t^{1-\alpha} \quad \text{...(15)}$$

$$\dot{K} = Y - C \quad \text{...(16)}$$

$$\dot{A} = \delta A L_A \quad \text{...(17)}$$

$$L = \bar{L} = L_Y + L_A \quad \text{...(18)}$$

The current-value Hamiltonian is:

$$\mathrm{H} = \frac{C^{1-\sigma}}{1-\sigma} + \theta_1\left[K^{\alpha}\left(L - L_A\right)^{1-\alpha} - C\right] + \theta_2\left[\delta A L_A\right]$$

The two decision variables are C_t and L_{At} ,so the first-order conditions are

$$\frac{d\mathrm{H}}{dC} = 0 \quad \text{...(19)}$$

$$\frac{d\mathrm{H}}{dL_A} = 0 \quad \text{...(20)}$$

and the co-state equations are:

$$\frac{d\mathrm{H}}{dK} = \rho\theta_1 - \dot{\theta}_1 \quad \text{...(21)}$$

$$\frac{d\mathrm{H}}{dL_A} = \rho\theta_2 - \dot{\theta}_2 \quad \text{...(22)}$$

Work out (19), $\frac{d\mathrm{H}}{dC} = 0 \Leftrightarrow C^{-\sigma} = \frac{\dot{C}}{C} = \frac{1}{\sigma}\frac{\dot{\theta}_1}{\theta_1}$,(20) $\frac{d\mathrm{H}}{dL_A} = 0 \Leftrightarrow \theta_1\left(1-\alpha\right)\left(L - L_A\right)^{-\alpha} K^{\alpha} A^{1-\alpha} = \theta_2 \delta A$

At (21), $\frac{d\mathrm{H}}{dK} = \rho\theta_1 - \dot{\theta}_1 \Leftrightarrow \frac{\dot{\theta}_1}{\theta_1} = \rho - \frac{\alpha\left(L - L_A\right)^{1-\alpha} A^{1-\alpha}}{K^{1-\alpha}}$,

(22) $\frac{d\mathrm{H}}{dA} = \rho\theta_2 - \dot{\theta}_2 \Leftrightarrow \theta_1\left(1-\alpha\right)\left(L - L_A\right)^{1-\alpha} K^{\alpha} A^{-\alpha} + \theta_2 \delta L_A = \rho\theta_2 - \dot{\theta}_2$

We solved model on its balanced growth path, the solution for which Y, K, and C grow at a constant rate (given by the growth rate of A), and the current-value prices θ_1 and θ_2 decline at constant rates. The first step is to look at (20) and (22) and view the similarity between their first terms. Combining these two equations give us:

$$\frac{\theta_2 \delta A\left(L-L_A\right)}{A}+\theta_2 \delta L_A=\rho\theta_2-\dot{\theta}_2 \Leftrightarrow \frac{\dot{\theta}_2}{\theta_2}=\rho-\delta L \quad \text{...(23)}$$

Next, log-differentiation in (20) direct to:

$$\frac{\dot{\theta}_1}{\theta_1}+\alpha\frac{\dot{K}}{K}+\left(1-\alpha\right)\frac{\dot{A}}{A}=\frac{\dot{\theta}_2}{\theta_2}+\frac{\dot{A}}{A} \Leftrightarrow \frac{\dot{\theta}_1}{\theta_1}=\frac{\dot{\theta}_2}{\theta_2} \quad \text{...(24)}$$

Now, equations (19), (23) and (24) are used to obtain the equilibrium growth rate of centralized problem:

$$g=\frac{\dot{C}}{C}=\frac{1}{\sigma}\frac{\dot{\theta}_1}{\theta_1}=-\frac{1}{\sigma}\frac{\dot{\theta}_2}{\theta_2} \Leftrightarrow gSP=\frac{\delta L-\rho}{\sigma} \quad \text{...(25)}$$

The centralized equilibrium growth rate, gSP, given by (25) is higher than the decentralized equilibrium growth rate, gD, given by (13), recurring, for better judgment:

$gD=\frac{\alpha\delta\bar{L}-\rho}{\alpha+\sigma}$, proving that Romer's decentralized model conveys a sub-optimal solution.

CONCLUSION

Today, the Romer (1990) model is central to study the economic growth. The model filled gap in the literature and enhances economists' considerate an endogenous technological change. The model integrates market-driven mechanism of innovation endogenously explain sustained technological change. However, given its complexity and difficult to demonstrate how the level of R&D is determined in the general market equilibrium within the Romer model.

This paper provides simplified way to understand the Romer model by presenting both consumer´s and production sides of the economy, the letter is made up of three productive sectors: final goods, producer durables and R&D sectors. We also derive step by step all sectors but not too far ahead in entire advanced mathematical calculus and algebra for the intuition purpose. Thus, this layout sheds light on the Romer model and become more accessible to enhance the theory of the knowledge- based economy. In line with our analysis, the further research should test the model by applying time series data against predictive parameters calibrated in the model.

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