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A CAPACITATED MODEL FOR A FINANCIALLY DISTRESSED COMMODITY TRADER MAKING SIMULTANEOUS OPERATIONAL AND FINANCIAL DECISIONS

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Abstract

This paper studies a commodity trader's problem of investing in transportation capacity for hopes of arbitrage opportunity. In this model setting the investment of transportation capacity is limited, and the commodity trader is facing risk due to financial distress cost. We analyze settings regarding the arbitrage opportunity of capacity pricing. Besides two trivial problem settings (risk neutral and mispriced high), this study shows transportation capacity investment is limited for settings, in which there are arbitrage opportunities (mispriced low and two measures).

Keywords:Capacity Management, Operational Hedging, Supply Chain Management, Operations Management, Commodity Transportation

INTRODUCTION

To motivate this study, we consider a commodity producer (or trader) located in Market 1, which is the domestic spot market, that buys transportation capacity in order to ship the commodity to another market, i.e., Market 2. We assume to production level being zero, hence the commodity producer is regarded as a trader, who invests on the transportation capacity if there is a value to ship the commodity to the Market 2. Moreover, by assuming the Market 2's spot market price is being constant; then, the domestic spot market price, Market 1, results the uncertainty in the decision-making problem. Therefore, the profit of the commodity trader in a single-period problem is stochastic and this leads to the financial distress cost for the trader, which is - well identified and researched in the finance literature - the risk associated with the bankruptcy cost and the corporate taxes of the firm.

This paper analyzes a capacitated model for the commodity trader, who is value-maximizer or risk-averse. This difference is caused by the upper bound on how much the trader buys the transportation capacity to ship the commodity from Market 1 to Market 2. This problem approach is investigated in four settings based on the measure and pricing of the transportation capacity: risk neutral, mispriced low, mispriced high, and two measures. The main research question for the commodity trader in Market 1 is about the structure of the optimal transportation capacity investment policy. How does the optimal policy change with respect to the parameters of the model? In this work, we characterize the optimal investment decision and also perform comparative static analysis for the commodity trader.

Natural gas producers and the local distribution companies in the transportation market for the natural gas market in US are the motivation behind this study (Birge, 2000; Civelek, 2014). The number of studies incorporating risk in the operations management is limited and also focused in the mean-variance analysis context; thus, the main contribution of this study is using more relevant evaluation of the trader instead of using the utility theory. This paper also incorporates the financial distress cost into the trader's evaluation in order to represent the risk associated with bankruptcy costs and corporate taxes. This allows us to show the impact of this risk in the structure of the optimal capacity investment decisions.

Section 2 reviews the related literature and places the contribution of this study. Model setting is presented in Section 3; then, the analysis of the problem for value maximizer trader and risk averse trader are provided in Section 4 and 5, respectively. Section 6 concludes this work with discussion and future research.

LITERATURE REVIEW

Flexibility in production processes is defined as the hedge against diversity in general terms (De Groote, 1994). On the other hand, risk analysis and operations management use the operational hedging concept as a flexibility. The operational hedging is defined as mitigating risk by using financial instruments (Chod et al., 2010). This study uses operational flexibility to create value to let transporting the commodity to Market 2 in hopes of selling the commodity with a higher price. Therefore, the commodity trader can get value by investing on the transportation capacity, which is an operational hedging tool against the uncertainty about the Market 1's spot market price.

Chen and Federgruen (2000) review the inventory management literature by using the mean-variance analysis and maximizing the expected utility functions. Eeckhoudt et al. (1995) show the optimal inventory level of a single product newsboy decreases with risk aversion. In addition to those studies in literature, Anvari (1987), Boukaiz and Sobel (1992), and Gaur and Seshadri (2005) study the problem of capacity investment under risk aversion. The main characteristic of this study separates it from papers from the literature is not using the mean-variance analysis.

In the finance literature, the financial hedging has been extensively studied and used in recent operations management studies. Diamond (1984) states that the bankruptcy costs of a firm lead to hedging and emphasizes the importance of diversification in spite of the risk neutrality. Diamond (1984) also shows that small firms are more likely to hedge and the hedging reduces the probability to incur bankruptcy costs, and shareholders benefit from hedging because the bankruptcy brings real costs to them. Moreover, Smith and Stulz (1985) present the financial distress costs and corporate taxes as the reason of the risks of a firm. This study models the financial distress cost provided by Brown and Toft (2002), in which the only thing the value maximizer trader can do is financial hedging to avoid risky states of his or her financial distress costs.

In light of capacity management literature incorporating risk, Birge (2000) used option pricing as a market hedge to incorporate risk into planning models by modifying capacity and resource levels. In this paper, we use similar exchanges of commodities like natural gas, but our study differentiates in using risk averse trader and two measure case for the price of the commodity.

MODEL

Notation used in this study:

c_1: Parameter that determines the overall effect of financial distress cost

c_2: Parameter that controls the curvature of the financial distress cost

s_1: Initial spot market price of Market 1

μ: Mean of the normally distributed $ln\ s_1$

σ: Standard deviation of the normally distributed $ln\ s_1$

Δt: Time horizon length of the single period

u: The rate for increased s_1

d: The rate for decreased s_1, $0<d<1$

s_{1H}: High value that s_1 can get at the end of one period, $s_{1H} = ln\ us_1$

s_{1L}: Low value that s_1 can get at the end of one period, $s_{1H} = ln\ ds_1$

S_2: Fixed price of the second market

p^*: Probability that s_1 will become s_{1H} in risk neutral measure, Q

p: Probability that s_1 will become s_{1H} in the other measure (trader's belief)

q: Capacity investment decision (decision variable)

$\bar{q}$: Upper bound on the capacity investment

K: Unit price of the capacity

$P(q,s_1)$: Profit at the end of the single period

$C(P(q,s_1))$: Financial distress cost at the end of the single period

$\pi(q)$: Expected profit at the end of the single period

Q: Risk-neutral probability measure

q_u: Upper bound on the transportation capacity

This paper uses the modeling approach using two different traders: Value maximizer and risk averse. In considering the trader's problem, the model directly incorporates the value of the trader's firm instead of using a utility function. The value maximizer trader maximizes the expected value of the firm, which is the expected cash flow minus expected financial distress cost. As for the risk averse trader, he or she focuses on maximizing the utility. We assume a risk-averse manager, whose compensation depends on a constant percentage of the firm's profit at the end of the single trading period. Additionally, his or her mission is to mitigate the impact of low profit states as much as possible because of his or her utility. For each trader type, the present study analyzes four cases based on the probability measure and the pricing of the transportation capacity: (i) risk neutral, (ii) mispriced low, (iii) mispriced high, and (iv) two measures.

There are two markets: Market 1, at where the trader buys and sells the commodity, and Market 2, at where the trader can ship the commodity and sell. The commodity trader buys the transportation capacity and also buys from Market 1 based on the spread among the random spot market price in Market 1 and the fixed price in Market 2. Regarding the single-period options pricing theory and the traditional assumptions on the stock prices, the uncertain commodity price, s_1 has a lognormal distribution: $\ln s_1$ is normally distributed with mean μ and standard deviation σ. σ refers to the volatility of the price. By the binomial options pricing theory, we select the time horizon of the single period problem, Δt, small enough (in our numerical studies, we let one day as the length of the single period trading time horizon, so $\Delta t = 1/365$). Moreover, in order to match the mean log return and the variance of the price, we use the standard model for u and d: $u=e^{\sigma\sqrt{\Delta t}}$ and $d=1/u$. Since the only uncertainty arises from s_1, we assume a two-point discrete distribution for s_1: $s_1 = s_{1H}$ with probability p and s_{1L} with probability $1-p$.

In the rest of the paper, we use s_{1H} = *ln* us_1 and s_{1L} = *ln* ds_1. We also assume $s_{1H} > S_2 > s_{1L}$ to avoid trivial situations in the problem setting. Moreover, the expected future price of the spot Market 1 is equal to the initial prices under risk neutral pricing of the transportation capacity. By assuming the risk-free rate being zero, the probability of the high-price state, s_{1H}, is $p^* = (1-d)/(u-d)$. By assumption, the unit price of the transport capacity, K, is determined by the spread of the price between two market prices under the risk-neutral measure, Q:

$$K=E_Q\left[\{S_2-s_1\}^+\right]=(1-p^*)\{S_2-s_{1L}\} \quad (1)$$

The profit position at the end of the single period is $P(q,s_1)=q(\{S_2-s_1\}^+-K)$.Hence, the corresponding financial distress cost regarding the profit in the future is $C(P(q,s_1)) = c_1e^{-c_2P(q,s_1)} = c_1e^{-c_2q(\{S_2-s_1\}^+-K)}$, where $c_1>0$ and $c_2>0$. Then, the expected profit of the firm is $\pi(q)=E[P(q,s_1)-C(P(q,s_1))]$.

Lemma 1. *The expected value of the firm, π(q), is concave.*

VALUE MAXIMIZER TRADER

In this section, we characterize the structure the optimal capacity investment decision of the value maximizer trader and perform an analysis of the comparative statics. In analyzing the behavior of the value function and analyzing the comparative statics, the following values are used for the parameters unless the change is specifically stated: $c_1=0.1$, $c_2=3$, $s1=2$, $S_2=\ln 2.5$, $\mu=0.15$ and $\sigma=0.20$. Moreover, the unit capacity price, K, *is 0.1* in the mispriced low and $1.5(1-p^*)(S_2-s_{1L})$ in the mispriced high.

In the literature using operational flexibility and hedging, the models are based on the exponential utility functions and mean-variance maximizer manager. Hence, the main distinguishing feature of our model is using the expected value of the firm directly without using any utility function. In the model, the firm's problem is:

$$\max_q \pi(q)=E\left[P(q,s_1)-C(P(q,s_1))\right] \quad (2)$$

Risk neutral

In this case, there is only one measure, which is risk neutral. Then, the expected profit of the firm is $E_QP(q,s_1)=K-K=0$. Therefore, the firm's problem is $max_q\,\pi(q) = E[-C(P(q,s_1))] = min_q\,E[-C(P(q,s_1))]$, subject to $q \le \bar{q}$ and $q \ge 0$.

Proposition 1. *The optimal transportation capacity investment, q*, is zero.*

By Proposition 1, the commodity trader never enters the transportation capacity business if there exists only risk neutral measure. With only risk neutral measure, there is no incentive for the trader to buy the transportation capacity; therefore, the solution is trivial.

Mispriced low

The price of the capacity is lower than its value under risk neutral measure: $K < E_Q[\{S_2\text{-}s_1\}^+]$.

Proposition 2. *Let $\hat{q}$, is the solution of the following first-order condition:* $(1-p^*)(S_2-s_{1L})-K-c_1c_2\{p^*Ke^{c_2qK}-(1-p^*)(S_2-s_{1L}-K)e^{-c_2q(S_2-s_{1L}-K)}\}=0$.*Then, the optimal transportation capacity decision, $q^*>0$, is equal to $\hat{q}$ if $\hat{q}<\bar{q}$, or $\bar{q}$ if $\hat{q}\geq\bar{q}$.*

Proposition 2 shows that the trader gets value for investment is the capacity is mispriced low. This value is, in fact, a pure arbitrage opportunity in the market since the commodity trader is buying from the Market 1 and selling it in the Market 2.

Proposition 3. *In the comparative statics, the following holds: (1) $\partial q^*/\partial c_1<0$, (2) $\partial q^*/\partial c_2<0$, (3)$\partial q^*/\partial K<0$, (4) $\partial q^*/\partial S_2>0$, (5) $\partial q^*/\partial s_1<0$ and (6) $\partial q^*/\partial\sigma<0$.*

The value maximizer trader decreases its investment as the impact of the financial distress cost, c_1 and c_2, increase. This result is intuitive because the firm invests less as the impact of financial distress cost increases. In order to understand the impacts of K, S_2, s_1 and σ, we analyze the variance of the profit at the end of single period:

$$Var[P(q,s_1)]=q^2\{p^*K^2+(1-p^*)(S_2-s_{1L}-K)^2-[(1-p^*)(S_2-s_{1L})-K]^2\} \qquad (3)$$

Proposition 4. *The impact of $E[P(q,s_1)]$ dominates the comparative statics of K, S_2, s_1 and σon the optimal investment, q*, instead of $Var[P(q,s_1)]$. Thus, the following holds:*

	$\partial q/.$	$\partial E_Q[P(q,s_1)]/.$	$\partial Var[P(q,s_1)]/.$
K	*<0*	*<0*	*0*
S_2	*>0*	*>0*	*<0*
s_1	*<0*	*<0*	*<0*
σ	*>0*	*>0*	*>0*

Regarding the managerial insights for the unit transportation capacity price *K*, the fixed Market 2's price, s_2, the initial price of the Market 1, s_1, and the volatility of s_1, σ, the value maximizer trader buys more on transportation capacity if the trader makes more profit on expectation whether there exists high or low risk caused by the variance of the profit. In other words, if s_2 or σ increases, the value maximizer trader earns more profit on expectation and buys more transportation capacity although the variance of the profit increases. On the contrary, if *K* or s_1 increases, the value maximizer firm makes less money on expectation; thus, the trader invests less on the transportation capacity in spite of decreasing of the variance of the profit. Therefore, making more profit on expectation is more significant on the value-maximizer firm's decisions regardless on the risk associated with the variance of the profit.

Mispriced high

The transportation capacity is priced high as$K > E_Q[\{S_2\text{-}s_1\}^+]$.

Proposition 5. *The optimal transportation capacity investment decision, q*, is zero.*

By Proposition 5, the commodity trader never enters the transportation capacity business if the capacity is mispriced high, in which selling the capacity enables the producer pure arbitrage. Consequently, this case of the value maximizer trader has a trivial solution, $q^*=0$.

Two-measures

In this case, the capacity is priced fairly by the risk neutral measure and the expected profit is measures by the firm's belief, which is different than the risk neutral measure. In order to avoid trivial solution, we assume $p^* > p$:

$$E[P(.)] = q\{-pK + (1-p)(S_2 - s_{1L} - K)\} = q(S_2 - s_{1L})(p^* - p) > 0 \Rightarrow p^* > p$$

Proposition 6. *Let* $\hat{q}$, *is the solution of the following first-order condition:*$p^* - p - c_1 c_2\left[p(1-p^*)e^{c_2(1-p^*)(S_2 - s_{1L})q} - (1-p)p^* e^{-c_2 p^*(S_2 - s_{1L})q}\right] = 0$.*Then, the optimal transportation capacity decision,* $q^* > 0$, *is equal to* $\hat{q}$ *if* $\hat{q} < \bar{q}$, *or* $\bar{q}$ *if* $\hat{q} \geq \bar{q}$.

By Proposition 6, the trader maximizes its expected profit by investing on transportation capacity, because the fair price by risk-neutral measure is lower than the firm's belief on the price, $p^* > p$: $(1-p^*)(S_2 - s_{1L}) < (1-p)(S_2 - s_{1L})$. Therefore, the price of the capacity is low according to the firm's belief and capacity investment creates value for the firm on expectation.

Proposition 7. *In the comparative statics, the following holds: (1)* $\partial q^*/\partial c_1 < 0$*, (2)* $\partial q^*/\partial c_2 < 0$*, (3)* $\partial q^*/\partial p < 0$*, (4)* $\partial q^*/\partial S_2 < 0$*, (5)* $\partial q^*/\partial s_1 > 0$ *and (6)* $\partial q^*/\partial \sigma < 0$*.*

The value maximizer trader invests less as the impact of the financial distress cost increases, c_1 and c_2 increase. This result is intuitive since the trader invests less as the impact of the financial distress cost increases. Additionally, the transportation capacity investment approaches to zero as p gets closer to the risk neutral probability, p^*, at where the problem becomes the case with one measure and the solution is trivial as $q^*=0$.

Proposition 8. *The impact of* $Var[P(q,s_1)]$ *dominates the comparative statics of* S_2*,* s_1 *and* σ*on the optimal investment,* q^**, instead of* $E[P(q,s_1)]$*. The following holds:*

	$\partial q/.$	$\partial E_Q[P(q,s_1)]/.$	$\partial Var[P(q,s_1)]/.$
S_2	<0	>0	>0
s_1	>0	<0	<0
σ	<0	>0	>0

The value maximizer firm invests more on transportation capacity if the variance of the profit decreases, because the firm makes more profit in expectation. This result is counter-intuitive regarding the case where the transportation capacity is low mispriced. In the comparative statics of S_2 and σ, the value-maximizer firm invests less on capacity if S_2 and σ increases in spite of expected profit improvement in both cases. The reason is that the variance of the profit decreases as S_2 and σ increases. Therefore, the impact of variance of the profit dominates the effect of the expected profit on the optimal transportation capacity investment, in which there are different measures: risk-neutral and value-maximizer firm's belief. Moreover, the risk associated with the variance of the profit is more significant than the expected profit on the capacity investment decision.

RISK AVERSE TRADER

In this study's problem setting, the risk averse trader's compensation depends on a certain percentage of the expected profit at the end of the single trading period, $\gamma P(.)$ where γ is the predetermined ratio of the trader's payoff. Since the firm has a risk associated with the expected financial distress cost, the risk-averse trader's utility also consists of this cost, *U(.) = E[-C(γP(q,s₁)]*. Thus, the risk-averse trader's problem is

$$\max_q E\left[-C\left(\gamma P(q,s_1)\right)\right] = \min_q E\left[C\left(\gamma P(q,s_1)\right)\right].$$

Since γ is a scale factor for the *P(.)*, we assume $\gamma=1$ for the rest of the paper. Hence, the risk-averse trader's problem is

$$\min_q E\left[C\left(P(q,s_1)\right)\right].$$

In the following subsections, we analyze 4 setting for the risk averse trader: risk-neutral, mispriced low, mispriced high, and two measures.

Risk neutral

This case is similar to the value maximizer trader's problem, in which the solution is trivial, $q^*=0$. This result is intuitive since there is no arbitrage opportunity for the trader in using only risk neutral measure, in which the capacity is already fairly priced.

Mispriced low

This case provides arbitrage opportunity for the trader, in which the price of the capacity is lower than its value under risk-neutral measure. In other words, the capacity is low-mispriced: $K < E_Q[\{S_2-s_1\}^+] \rightarrow K < (1-p^*)(S_2 - s_{1L})$.

Proposition 9. *Let* $\hat{q},\hat{q} = \frac{1}{c_2(S_2-s_{1L})}\ln\left[\frac{(1-p^*)(S_2-s_{1L}-K)}{p^*K}\right]$. *Then, the optimal investment decision,* $q^* > 0$*, is equal to* $\hat{q}$ *if* $\hat{q} < \bar{q}$*, or* $\bar{q}$ *if* $\hat{q} \geq \bar{q}$*.*

Since we assume only exponential utility function in this case, there is a closed form solution for q^*. Before the comparative statics of the optimal investment decision, we first define *mispricing ratio,* ψ*.*

Definition 1. *The mispricing ratio,* ψ*, is used to quantify the low mispricing of the unit capacity price, K, regarding the risk-neutral measure, Q, and it is defined as*

$$\psi = \frac{K}{(1-p^*)(S_2-s_{1L})},$$

Where, $0 < \psi < 1$ *by definition.*

Proposition 10. *Let $\hat{\psi}$ be the critical value for the mispriced ratio, at where comparative statics of S_2 and s_1 change. Additionally, $\widehat{S_2}$ and $\widehat{s_1}$ are the critical values of S_2 and s_1 depending on $\hat{\psi}$.*

$$\hat{\psi} = \frac{1}{(1-p^*)} \frac{LambertW\left(\frac{1-p^*}{p^* e}\right)}{1 + LambertW\left(\frac{1-p^*}{p^* e}\right)}$$

$$\widehat{S_2} = s_{1L} + \frac{K}{\hat{\psi}(1-p^*)}$$

$$\widehat{s_1} = \frac{1}{d} e^{S_2 - \frac{K}{\hat{\psi}(1-p^*)}}.$$

Then, the following holds in the comparative statics:

(i) $\partial q / \partial c_2 < 0$

(ii) $\partial q / \partial K < 0$

(iii) $\partial q / \partial \sigma < 0$

(iv) $$\begin{cases} \partial q/\partial S_2 > 0 & , s_{1L} + \frac{K}{1-p^*} < S_2 < \widehat{S_2} \\ \partial q/\partial S_2 = 0 & , S_2 = \widehat{S_2} \\ \partial q/\partial S_2 < 0 & , \widehat{S_2} < S_2 < s_{1H} \end{cases}$$

(v) $$\begin{cases} \partial q/\partial s_1 > 0 & , s_{1L} < s_1 < \widehat{s_1} \\ \partial q/\partial s_1 = 0 & , s_1 = \widehat{s_1} \\ \partial q/\partial s_1 < 0 & , \widehat{s_1} < s_1 < \frac{1}{d} e^{S_2 - \frac{K}{(1-p^*)}} \end{cases}$$

Considering the comparative statics of the case for the value-maximizer firm, Proposition 3, the impacts of c_2, K and σ are similar for the risk-averse trader. Hence, the risk averse trader also invests less on the transportation capacity as the effect of financial distress cost increases or the unit price of the capacity increases. However, he or she invests more on capacity as σ increases. Moreover, the impact of S_2 and s_1 depend on both K and ψ. Therefore, the impact of s_1 depends on both K and S_2.

Mispriced high

The capacity is high-mispriced: $K > E_Q[\{S_2 - s_1\}^+] \rightarrow K > (1-p^*)(S_2 - s_{1L})$.

Proposition 11. *The optimal transportation capacity decision, q*, is zero.*

The risk averse trader never enters the transportation capacity investment business if the capacity is mispriced high. Similar to the value-maximizer firm, the solution is trivial, $q^*=0$.

Two measures

The transportation capacity is fairly priced by the risk neutral measure of the shipper; however, the value of the shipping capacity is measured by the risk averse trader's measure. In order to avoid the trivial solution, *p*>p* is assumed. We also define a ratio called the *likelihood* of the increase of the price for each measure: *L**and *L*.

Definition 2. *$L^*=p^*/(1-p^*)$ and $L=p/(1-p)$ are the likelihood of an increase, upward movement, of the spot market price in market 1, s_1, in the risk-neutral measure and the other measure (firm's belief), respectively.*

Proposition 12. *Let $\hat{q}$, $\hat{q} = \frac{1}{c_2(S_2 - s_{1L})} \ln\left(\frac{L^*}{L}\right)$. Then, the optimal investment decision, $q^* > 0$, is equal to $\hat{q}$ if $\hat{q} < \bar{q}$, or $\bar{q}$ if $\hat{q} \geq \bar{q}$.*

In the closed form solution of q^*, L^* is always greater than *L,* due to the assumption of $p^*>p$. Therefore, the risk averse trader buys capacity if the likelihood of the trader's belief, *L,* is less than the risk neutral measure, L^*.Considering the case for the risk averse trader and value maximizer trader, the comparative statics are the same. Thus, the risk averse trader invests less on the capacity as the effect of the financial distress cost, c_2, increases. Moreover, the risk averse trader invests less on the capacity as the trader's belief about the probability on the upward movement in s_1, *p,* increases. In other words, the transportation capacity investment becomes closer to zero as *p* gets closer to the risk-neutral probability, p^*, in which the problem becomes the case with one measure and has a trivial solution, $q^*=0$. As for analyzing the impacts of S_2, s_1 and σ, we analyze that via Proposition 8.

CONCLUSION

In this commodity trading model by the marketing flexibility with the financial distress cost, the trader invests to the maximum amount if shipping the commodity to the secondary market is valuable. We prove that there is a trivial solution, $q^* = 0$, in the case in where only risk neutral measure and the mispriced high cases. As for the managerial insights regarding mispriced low, the trader does not invest infinitely on the capacity even if there might be a pure arbitrage opportunity. The intuition is that the financial distress cost becomes very significant and the trader will start to hurt himself or herself after a large transportation capacity investment; therefore, the variance of the profit and the financial distress cost clearly prevent the trader from investing infinite amount of capacity. Focusing on the comparative statics of s_1, S_2 and σ, the impacts of the profit on expectation and the variance of the profit change in the mispriced low case. In determining the comparative static of s_1, S_2 and σ, the profit on expectation dominates the variance in the mispriced low case.

The implications of this study are that the commodity trader needs to determine both operational and financial decisions simultaneously in order to mitigate risk from spot market prices and uncertain demand conditions. Managers in commodity production and trading companies, i.e., natural gas, oil and coal producers/traders, need to incorporate operational and financial decisions simultaneously in order to mitigate risk from spot market prices. Considering mispriced low case for the transportation capacity, the commodity trader does not invest infinitely on the transportation capacity even if a pure arbitrage opportunity may exist. This result provides an insightful managerial implication such that the financial distress cost becomes significant for the commodity trader.

For future research, risk averse commodity trader's problem or considerations of financial hedging decisions are promising research problems. There are also limitations of our study, in which commodity trading problems involve stochastic structure of commodity trading with transportation capacity investment and financial hedging in practice, Moreover, incentive limitations arise in the transportation of the commodities by commodity producer and traders in different markets.

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APPENDIX

Proof of Lemma 1. Since $P(q,s_1)$ is linear and $C(P(q,s_1))$ is convex in q, $P(q,s_1)-C(P(q,s_1))$ is concave. By the preservation of concavity under expectation, $\pi(q)$ is concave.

Proof of Proposition 1. $max_q\, \pi(q) = min_q\, c_1\{e^{c_2 q(1-p^*)(S_2-s_{1L})} + (1-p^*)e^{-c_2 q p^*(S_2-s_{1L})}\}$, *subject to* $q \geq 0$. KKT conditions are (i)$\lambda \geq 0$, (ii)$\lambda q^* = 0$and(iii)$c_1 c_2 p^*(1-p^*)(S_2-s_{1L})\{e^{c_2 q(1-p^*)(S_2-s_{1L})} - e^{-c_2 q p^*(S_2-s_{1L})}\} - \lambda = 0 \Rightarrow q^* = 0$ and$\lambda = 0$. Then, $q \geq 0$is weakly active. Since$E_Q[C(P(q,s_1))]$and$q \geq 0$ are convex, $q^*=0$ is the global optimum by KKT.

Proof of Proposition 2. $\max_q \pi(q) = q\{(1-p^*)(S_2-s_{1L}) - K\} - c_1\{p^* e^{c_2 qK} - (1-p^*)e^{-c_2 q(S_2-s_{1L}-K)}\}$, subject to $q \geq 0$. KKT conditions are (i)$\lambda \geq 0$, (ii)$\lambda q^* = 0$and (iii) $(1-p^*)(S_2-s_{1L}) - K - c_1 c_2\{p^* K e^{c_2 qK} - (1-p^*)(S_2-s_{1L}-K)e^{-c_2 q(S_2-s_{1L}-K)}\} + \lambda = 0 \Rightarrow q^* > 0$ and $\lambda = 0$. Then,$q \geq 0$is weakly active. Since$\pi(q)$ and$q \geq 0$are concave,$q^* > 0$is the global optimum by KKT. Therefore, the first order condition gives the global optimum.

Proof of Proposition 3. *(1)* $\partial q^*/\partial c_1 < 0$*:*

$$\frac{\partial q}{\partial c_1} = -\frac{-c_2\{p^* K e^{c_2 qK} - (1-p^*)(S_2-s_{1L}-K)e^{-c_2 q(S_2-s_{1L}-K)}\}}{-c_1 c_2{}^2\{p^* K^2 e^{c_2 qK} - (1-p^*)(S_2-s_{1L}-K)^2 e^{-c_2 q(S_2-s_{1L}-K)}\}}$$

$$\alpha_1 = p^* K^2 e^{c_2 qK} - (1-p^*)(S_2-s_{1L}-K)^2 e^{-c_2 q(S_2-s_{1L}-K)} > 0$$

$$\alpha_2 = p^* K e^{c_2 qK} - (1-p^*)(S_2-s_{1L}-K)e^{-c_2 q(S_2-s_{1L}-K)} = \frac{1}{c_1 c_2}\frac{\partial E_Q[C(P(q,s_1))]}{\partial q}$$

Since $E_Q[C(P(q,s_1))]$is strictly increasing in q, $\alpha_2 > 0$. By definition, $c_1>0$, $c_2>0$. Therefore, $\partial q^*/\partial c_1 = -\alpha_2/(c_1 c_2 \alpha_1) < 0$.

(2) $\partial q^*/\partial c_2 < 0$*:*

$$\frac{\partial q}{\partial c_2} = -\frac{-c_1 c_2(\alpha_2 + c_2\alpha_1)}{-c_1 c_2{}^2 \alpha_1} = -\frac{\alpha_2 + c_2\alpha_1}{c_2\alpha_1} < 0$$

(3) $\partial q^*/\partial K < 0$*:*

$$\frac{\partial q}{\partial K} = -\frac{-1 - c_1 c_2\{p^* K e^{c_2 qK}(1 + c_2 qK) + (1-p^*)[1 + c_2 qK(S_2-s_{1L}-K)]e^{-c_2 q(S_2-s_{1L}-K)}\}}{-c_1 c_2{}^2\{p^* K^2 e^{c_2 qK} + (1-p^*)(S_2-s_{1L}-K)^2 e^{-c_2 q(S_2-s_{1L}-K)}\}}$$

$$\alpha_3 = p^* K e^{c_2 qK}(1 + c_2 qK) + (1-p^*)[1 + c_2 qK(S_2-s_{1L}-K)]e^{-c_2 q(S_2-s_{1L}-K)} > 0$$

$$\Longrightarrow \frac{\partial q}{\partial K} = -\frac{1 + c_1 c_2 \alpha_3}{c_1 c_2^2 \alpha_1} < 0$$

(4) $\partial q^*/\partial S_2 > 0$*:*

$$\frac{\partial q}{\partial S_2} = \frac{(1-p^*)\{1 + c_1 c_2 e^{-c_2 q(S_2-s_{1L}-K)}[1 - c_2 q(S_2-s_{1L}-K)]\}}{c_1 c_2{}^2\{p^* K^2 e^{c_2 qK} + (1-p^*)(S_2-s_{1L}-K)^2 e^{-c_2 q(S_2-s_{1L}-K)}\}}$$

In the case of low-mispriced capacity, $K<(1-p^*)(S_2-s_{1L}) \rightarrow S_2>[K/(1-p^*)]+s_{1L}$. By using the nonnegativity constraint on q:

$$e^{-c_2 q(S_2-s_{1L}-K)}[1 - c_2 q(S_2-s_{1L}-K)] \to 1, q \to 0$$
$$e^{-c_2 q(S_2-s_{1L}-K)}[1 - c_2 q(S_2-s_{1L}-K)] \to 0, q \to \infty$$
$$\Longrightarrow 0 < e^{-c_2 q(S_2-s_{1L}-K)}[1 - c_2 q(S_2-s_{1L}-K)] < 1$$

Since $\alpha_2 < 0$, then $\partial q^*/\partial S_2 > 0$.
(5) $\partial q^*/\partial s_1 < 0$*:*

$$\frac{\partial q}{\partial \ln s_1} = -\frac{-\frac{(u-1)d}{u-d} - c_1c_2\left\{-\frac{(u-1)}{u-d}\left[-de^{-c_2q(S_2-\ln ds_1-K)} + c_2q(S_2-\ln ds_1-K)^2e^{-(S_2-\ln ds_1-K)}\right]\right\}}{-\frac{c_1c_2{}^2}{u-d}\{(1-d)K^2e^{c_2qK} + (u-1)(S_2-\ln ds_1-K)^2e^{-(S_2-\ln ds_1-K)}\}}$$
$$= \frac{(u-1)d\left\{1-c_1c_2e^{-c_2q(S_2-\ln ds_1-K)}[1-c_2q(S_2-\ln ds_1-K)]\right\}}{c_1c_2{}^2\{(1-d)K^2e^{c_2qK} + (u-1)(S_2-\ln ds_1-K)^2e^{-(S_2-\ln ds_1-K)}\}}$$

By using nonnegativity constraint on q:

$$e^{-c_2q(S_2-\ln ds_1-K)}[1-c_2q(S_2-\ln ds_1-K)] \to 1, q \to 0$$
$$e^{-c_2q(S_2-\ln ds_1-K)}[1-c_2q(S_2-\ln ds_1-K)] \to 0, q \to \infty$$
$$\Rightarrow 0 < e^{-c_2q(S_2-\ln ds_1-K)}[1-c_2q(S_2-\ln ds_1-K)] < 1$$
$$\Rightarrow -1-c_1c_2e^{-c_2q(S_2-\ln ds_1-K)}[1-c_2q(S_2-\ln ds_1-K)] < 0$$

Since $(1-d)K^2e^{c_2qK} + (u-1)(S_2-\ln ds_1-K)^2e^{-(S_2-\ln ds_1-K)} > 0$ and$\partial q/\partial \ln s_1 < 0$, then $\partial q/\partial s_1 = (\partial q/\partial \ln s_1)s_1 < 0$.
(6) $\partial q^*/\partial \sigma < 0$*:*
Let $\beta_2 = uKe^{c_2qK}(S_2 - \ln s_1 + \ln u - K)$ and $\partial q/\partial u$:

$$\frac{\partial q}{\partial u} = \frac{u(S_2-\ln s_1+\ln u-K) - c_1c_2\beta_2e^{-c_2q(S_2-\ln s_1+\ln u-K)}}{u+1}$$

Let α_4, α_5, α_6, β_3 and α_7:

$$\alpha_4 = u(S_2-\ln s_1+\ln u) - c_1c_2\beta_2e^{-c_2q(S_2-\ln s_1+\ln u-K)}$$
$$\alpha_5 = u(S_2-\ln s_1+\ln u+1) + c_1c_2\beta_2e^{-c_2q(S_2-\ln s_1+\ln u-K)}[(S_2-\ln s_1+\ln u-K)(1+c_2q)+1]$$
$$\alpha_6 = \frac{K^2e^{c_2qK}}{u+1} + \frac{u}{u+1}(S_2-\ln s_1+\ln u-K)^2e^{-c_2q(S_2-\ln s_1+\ln u-K)} > 0$$
$$\beta_3 = Ke^{c_2qK} + e^{-c_2q(S_2-\ln s_1+\ln u-K)}[(S_2-\ln s_1+\ln u-K)(1+c_2q-u)+1]$$
$$\alpha_7 = \frac{S_2-\ln s_1+\ln u+u+1+c_1c_2\beta_3}{(u+1)^3} > 0$$

By substituting α_4, α_5, α_6 and α_7 into $\partial q/\partial u$:

$$\frac{\partial q}{\partial u} = -\frac{-\frac{\alpha_4}{(u+1)^2} + \frac{\alpha_5}{u+1}}{-c_1c_2^2\alpha_6} = \frac{\alpha_7}{c_1c_2^2\alpha_6} > 0$$

By expanding *u* as $u = e^{\sigma\Delta t}$ and the chain rule:

$$\frac{\partial q}{\partial u} = \frac{\partial q}{\partial u}\Delta te^{\sigma\Delta t} > 0, \Delta t > 0$$

Proof of Proposition 4. We will show $\partial q/.$, $\partial E_Q[P(q,s_1)]/.$ and $\partial Var[P(q,s_1)]/.$ for *K*, *S₂*, *s₁*and *σ*. First, $E_Q[P(q,s_1)]$, $E_Q[P^2(q,s_1)]$ and $Var[P(q,s_1)]$ are the following:

$$E_Q[P(q,s_1)] = q[(1-p^*)(S_2-s_{1L}-K)]$$
$$E_Q[P^2(q,s_1)] = q^2[p^*K^2 + (1-p^*)(S_2-s_{1L}-K)^2]$$
$$Var[P(q,s_1)] = q^2\{p^*K^2 + (1-p^*)(S_2-s_{1L}-K)^2 - [(1-p^*)(S_2-s_{1L})-K]^2\}$$

(i) $\partial q/\partial K < 0$,$E_Q[P(q,s_1)]/\partial K < 0$ and $\partial Var[P(q,s_1)]/\partial K = 0$:
We proved in$\partial q/\partial K < 0$ Proposition 3. Then, $E_Q[P(q,s_1)]/\partial K$ and $\partial Var[P(q,s_1)]/\partial K$:

$$\frac{\partial E_Q[P(q,s_1)]}{\partial K} = -qK < 0$$
$$\frac{\partial Var[P(q,s_1)]}{\partial K} = q^2\left[2p^*K - 2(1-p^*)(S_2-s_{1L}-K) + 2\big((1-p^*)(S_2-s_{1L})-K\big)^2\right] = 0$$

(ii) $\partial q/\partial S_2 > 0$,$E_Q[P(q,s_1)]/\partial S_2 > 0$ and $\partial Var[P(q,s_1)]/\partial S_2 > 0$:
We proved $\partial q/\partial S_2 > 0$ in Proposition 3. Then, $E_Q[P(q,s_1)]/\partial S_2$ and $\partial Var[P(q,s_1)]/\partial S_2$:

$$\frac{\partial E_Q[P(q,s_1)]}{\partial S_2} = q(1-p^*) > 0$$
$$\frac{\partial Var[P(q,s_1)]}{\partial S_2} = 2q^2p^*(1-p^*)(S_2 - s_{1L}) > 0$$

(iii) $\partial q/\partial s_1 < 0, E_Q[P(q,s_1)]/\partial s_1 < 0$ and $\partial Var[P(q,s_1)]/\partial s_1 > 0$:
We proved $\partial q/\partial s_1 < 0$ in Proposition 3.

$$\frac{\partial E_Q[P(q,s_1)]}{\partial \ln s_1} = -q(1-p^*) < 0 \Rightarrow \frac{\partial E_Q[P(q,s_1)]}{\partial s_1} = \frac{\partial E_Q[P(q,s_1)]}{\partial \ln s_1}\frac{1}{s_1} < 0$$
$$\frac{\partial Var[P(q,s_1)]}{\partial \ln s_1} = -2q^2p^*(1-p^*)(S_2 - s_{1L}) < 0 \Rightarrow \frac{\partial Var[P(q,s_1)]}{\partial s_1} = \frac{\partial Var[P(q,s_1)]}{\partial \ln s_1}\frac{1}{s_1} < 0$$

(iv) $\partial q/\partial \sigma > 0, E_Q[P(q,s_1)]/\partial \sigma > 0$ and $\partial Var[P(q,s_1)]/\partial \sigma > 0$:
We proved $\partial q/\partial \sigma > 0$ in Proposition 3.

$$\frac{\partial E_Q[P(q,s_1)]}{\partial \ln u} = q(1-p^*) > 0 \Rightarrow \frac{\partial E_Q[P(q,s_1)]}{\partial \sigma} = \frac{\partial E_Q[P(q,s_1)]}{\partial \ln u}\frac{1}{u}\Delta t e^{\Delta t} > 0$$

Let β_4 and β_5:

$$\beta_4 = (2-p^*)(S_2 - s_{1L} - K)^2 - \frac{K^2}{u-d} > 0$$
$$\beta_5 = \frac{2p^*(u-1)(S_2 - s_{1L} - K)}{u} - 2p^*(1-p^*)(S_2 - s_{1L} - K)(S_2 - s_{1L}) > 0$$

$$\frac{\partial Var[P(q,s_1)]}{\partial u} = \frac{q^2}{u-d}(\beta_4 + \beta_5) > 0$$

By expanding *u* as $u = e^{\Delta t}$ and the chain rule:

$$\frac{\partial Var[P(q,s_1)]}{\partial \sigma} = \frac{\partial Var[P(q,s_1)]}{\partial u}\Delta t e^{\Delta t} > 0$$

Proof of Proposition 5. The problem of the value-maximizer firm is:

$$\max_q \pi(q) = q[(1-p^*)(S_2 - s_{1L}) - K] - c_1\left[p^* e^{c_2 q(1-p^*)(S_2 - s_{1L})} + (1-p^*)e^{-c_2 q p^*(S_2 - s_{1L})}\right]$$
$$\Rightarrow \min_q \pi(q) = q[-(1-p^*)(S_2 - s_{1L}) + K] + c_1\left[p^* e^{c_2 q(1-p^*)(S_2 - s_{1L})} + (1-p^*)e^{-c_2 q p^*(S_2 - s_{1L})}\right]$$

subject to *q* ≥ *0*. KKT conditions are (i)$\lambda \geq 0$, (ii)$\lambda q^* = 0$and (iii) $K - (1-p^*)(S_2 - s_{1L}) + c_1c_2(1-p^*)(S_2 - s_{1L})\left\{e^{c_2 q(1-p^*)(S_2 - s_{1L})} - e^{-c_2 q p^*(S_2 - s_{1L})}\right\} - \lambda = 0 \Rightarrow q^* = 0$ and$\lambda = K - (1-p^*)(S_2 - s_{1L}) > 0$. Then, $q \geq 0$is strongly active. If the nonnegativity constraint is relaxed, there exists global minimum$q < 0$ by the concavity of $\pi(q)$ and KKT.

Proof of Proposition 6.
$\max_q \pi(q) = q(S_2 - s_{1L})(p^* - p) - c_1\left[p e^{c_2 q(1-p^*)(S_2 - s_{1L})} + (1-p)e^{-c_2 q p^*(S_2 - s_{1L})}\right]$ subject to $q \geq 0$. KKT conditions are (i)$\lambda \geq 0$, (ii)$\lambda q^* = 0$and (iii) $(S_2 - s_{1L})\left\{p^* - p - c_1c_2\left[p(1-p^*)e^{c_2 q(1-p^*)(S_2 - s_{1L})} + (1-p)p^* e^{-c_2 q p^*(S_2 - s_{1L})}\right]\right\} + \lambda = 0 \Rightarrow q^* \geq 0$ and$\lambda = 0$. Then, $q \geq 0$is weakly active. Since $\pi(q)$ and $q \geq 0$ are concave, $q^* \geq 0$ is the global optimum by KKT. Hence, the first order condition gives the global maximum.

Proof of Proposition 7. *(1)* $\partial q^*/\partial c_1 < 0$*:*

$$\frac{\partial q^*}{\partial c_1} = -\frac{-c_2\left[p(1-p^*)e^{c_2 q(1-p^*)(S_2 - s_{1L})} + (1-p)p^* e^{-c_2 q p^*(S_2 - s_{1L})}\right]}{-c_1c_2^2(S_2 - s_{1L})[p(1-p^*)^2 e^{c_2 q(1-p^*)(S_2 - s_{1L})} + (1-p)(p^*)^2 e^{-c_2 q p^*(S_2 - s_{1L})}]}.$$

Let

$$\alpha_8 = c_1c_2^2(S_2 - s_{1L})\left[p(1-p^*)^2 e^{c_2 q(1-p^*)(S_2 - s_{1L})} + (1-p)(p^*)^2 e^{-c_2 q p^*(S_2 - s_{1L})}\right] > 0$$

$$\alpha_9 = p(1-p^*)e^{c_2q(1-p^*)(S_2-s_{1L})} + (1-p)p^*e^{-c_2qp^*(S_2-s_{1L})} = \frac{\partial E_Q[C(P(.))]}{\partial q}\frac{1}{c_1c_2(S_2-s_{1L})}.$$

Since $E_Q[C(P(.))]$ is strictly increasing in q, $\alpha_9 > 0$.

$$\frac{\partial q^*}{\partial c_1} = -\frac{\alpha_8}{\alpha_9} < 0.$$

(2) $\partial q^*/\partial c_2 < 0$:

Let $\alpha_{10} = p(1-p^*)^2qe^{c_2q(1-p^*)(S_2-s_{1L})} + (1-p)(p^*)^2qe^{-c_2qp^*(S_2-s_{1L})} > 0$, then

$$\frac{\partial q^*}{\partial c_2} = -\frac{-c_1[\alpha_9 + c_2(S_2-s_{1L})\alpha_{10}]}{-\alpha_8} < 0.$$

(3) $\partial q^*/\partial p < 0$:

$$\frac{\partial q^*}{\partial p} = -\frac{-1-c_1c_2\left[(1-p^*)e^{c_2q(1-p^*)(S_2-s_{1L})} + p^*e^{-c_2qp^*(S_2-s_{1L})}\right]}{-c_1c_2^2(S_2-s_{1L})\alpha_{10}}.$$

Let $\alpha_{11} = (1-p^*)e^{c_2q(1-p^*)(S_2-s_{1L})} + p^*e^{-c_2qp^*(S_2-s_{1L})} > 0$, then

$$\frac{\partial q^*}{\partial p} = -\frac{-1-c_1c_2\alpha_{11}}{-c_1c_2^2(S_2-s_{1L})\alpha_{10}} < 0.$$

(4) $\partial q^*/\partial S_2 < 0$:

$$\frac{\partial q^*}{\partial S_2} = -\frac{q}{(S_2-s_{1L})} < 0.$$

(5) $\partial q^*/\partial s_1 > 0$:

Let $\beta_6 = p(1-p^*)^2e^{c_2q(1-p^*)(S_2-\ln s_1-\ln d)} + (1-p)(p^*)^2e^{-c_2qp^*(S_2-\ln s_1-\ln d)}$, then

$$\frac{\partial q^*}{\partial \ln s_1} = -\frac{-c_1c_2^2q\beta_6}{-s_1c_1c_2^2(S_2-\ln s_1-\ln d)(-\beta_6)} = \frac{q}{s_1(S_2-\ln s_1-\ln d)} > 0$$

$$\frac{\partial q^*}{\partial s_1} = \frac{\partial q^*}{\partial \ln s_1}\frac{1}{s_1} > 0.$$

(6) $\partial q^*/\partial \sigma < 0$.

Let α_{12}, α_{13} and α_{14} are the following:

$$\alpha_{12} = pe^{\frac{u}{u+1}c_2q(S_2-\ln s_1+\ln u)}\left[1+\frac{uq(S_2-\ln s_1+\ln u+u+1)}{u+1}\right] > 0,$$

$$\alpha_{13} = (1-p)e^{\frac{-1}{u+1}c_2q(S_2-\ln s_1+\ln u)}\left[1+q\frac{(u+u\ln u+1)}{u(u+1)}\right] > 0,$$

$$\alpha_{14} = p\frac{u^2}{(u+1)^2}e^{\frac{u}{u+1}c_2q(S_2-\ln s_1+\ln u)} + \frac{(1+p)}{(u+1)^2}e^{\frac{-1}{u+1}c_2q(S_2-\ln s_1+\ln u)} > 0.$$

Then,

$$\frac{\partial q^*}{\partial u} = -\frac{-\frac{1}{(u+1)^2}\left(1+c_1c_2^2(\alpha_{12}+\alpha_{13})\right)}{-c_1c_2^2(S_2-\ln s_1+\ln u)\alpha_{14}} < 0.$$

By substituting $u = e^{\sigma\Delta t}$ and the chain rule:

$$\frac{\partial q^*}{\partial \sigma} = \frac{\partial q^*}{\partial u}\Delta te^{\sigma\Delta t} < 0.$$

Proof of Proposition 8. *First, $E[P(q,s_1)]$, $E[P^2(q,s_1)]$and $Var[P(q,s_1)]$ are the following:*

$$E[P(q,s_1)] = q(S_2-s_{1L})(p^*-p),$$
$$E[P^2(q,s_1)] = q^2(S_2-s_{1L})^2[p(1-p^*)^2+(1-p)(p^*)^2],$$
$$Var[P(q,s_1)] = q^2(S_2-s_{1L})^2p(1-p).$$

(i) $\partial q/\partial S_2 < 0$, $\partial E[P(q,s_1)]/\partial S_2 > 0$ and $\partial Var[P(q,s_1)]/\partial S_2 > 0$:

$$\frac{\partial q}{\partial S_2} = -\frac{q}{S_2-s_{1L}} < 0,$$

$$\frac{\partial E[P(q,s_1)]}{\partial S_2} = q(p^* - p) > 0,$$
$$\frac{\partial Var[P(q,s_1)]}{\partial S_2} = 2q^2(S_2 - s_{1L})p(1-p) > 0.$$

(ii) $\partial q/\partial s_1 > 0$, $\partial E[P(q,s_1)]/\partial s_1 < 0$ and $\partial Var[P(q,s_1)]/\partial s_1 < 0$:

$$\frac{\partial q}{\partial \ln s_1} = \frac{q}{s_1(S_2 - \ln s_1 - \ln d)} > 0 \Rightarrow \frac{\partial q}{\partial s_1}\frac{1}{s_1} > 0,$$
$$\frac{\partial E[P(q,s_1)]}{\partial \ln s_1} = -q(p^* - p) < 0,$$
$$\frac{\partial E[P(q,s_1)]}{\partial s_1} = \frac{\partial E[P(q,s_1)]}{\partial \ln s_1}\frac{1}{s_1} < 0,$$
$$\frac{\partial Var[P(q,s_1)]}{\partial \ln s_1} = -2q^2(S_2 - s_{1L})p(1-p) < 0,$$
$$\frac{\partial Var[P(q,s_1)]}{\partial s_1} = \frac{\partial Var[P(q,s_1)]}{\partial \ln s_1}\frac{1}{s_1} < 0.$$

(iii) $\partial q/\partial \sigma > 0$, $\partial E[P(q,s_1)]/\partial \sigma > 0$ and $\partial Var[P(q,s_1)]/\partial \sigma > 0$:

$$\frac{\partial q}{\partial u} = -\frac{-\frac{u}{u+1}q[1 + c_1c_2^2(\alpha_{12} + \alpha_{13})]}{-c_1c_2^2(S_2 - \ln s_1 + \ln u)\alpha_{14}} < 0.$$

By substituting $u = e^{\sigma\Delta t}$ and the chain rule:

$$\frac{\partial q}{\partial \sigma} = \frac{\partial q}{\partial u}\Delta t e^{\sigma\Delta t} < 0.$$

Then,

$$\frac{\partial E[P(q,s_1)]}{\partial u} = q(p^* - p)\frac{1}{u} > 0,$$
$$\frac{\partial E[P(q,s_1)]}{\partial \sigma} = \frac{\partial E[P(q,s_1)]}{\partial u}\Delta t e^{\sigma\Delta t} > 0,$$
$$\frac{\partial Var[P(q,s_1)]}{\partial u} = 2q^2(S_2 - s_{1L})p(1-p)\frac{1}{u} > 0,$$
$$\frac{\partial Var[P(q,s_1)]}{\partial \sigma} = \frac{\partial Var[P(q,s_1)]}{\partial u}\Delta t e^{\sigma\Delta t} > 0.$$

Proof of Proposition 9.

$$\min_q c_1\left[p^* e^{c_2qK} + (1-p^*)e^{-c_2q(S_2 - s_{1L} - K)}\right], subject\ to\ q \geq 0.$$

KKT conditions are (i)$\lambda \geq 0$, (ii)$\lambda q^* = 0$and (iii) $c_1c_2\left[p^*Ke^{c_2qK} - (1-p^*)(S_2 - s_{1L} - K)e^{-c_2q(S_2 - s_{1L} - K)}\right] - \lambda = 0 \Rightarrow q^* > 0$ and$\lambda = 0$. Then, $q \geq 0$is weakly active. Since $E\left[C\left(P(q,s_1)\right)\right]$ and $q \geq 0$ are convex, $q^* > 0$ is the global optimum by KKT. Hence, the first order condition gives the global minimum.

Proof of Proposition 10.

(i) $\partial q/\partial c_2 < 0$

$$\frac{\partial q}{\partial c_2} = -\frac{1}{c_2^2(S_2 - s_{1L})}\ln\left[\frac{(1-p^*)(S_2 - s_{1L} - K)}{p^*K}\right]$$

Since $K < (1-p^*)(S_2 - s_{1L})$,

$$\ln\left[\frac{(1-p^*)(S_2 - s_{1L} - K)}{p^*K}\right] > 0$$

By assuming $S_2 > s_{1L}$, *then* $(S_2 - s_{1L}) > 0$. *Therefore,* $\partial q/\partial c_2 < 0$.

(ii) $\partial q/\partial K < 0$

$$\frac{\partial q}{\partial K} = -\frac{1}{c_2^2(S_2 - s_{1L})}\left(\frac{1}{S_2 - s_{1L} - K} + \frac{1}{K}\right)$$

Since $(S_2 - s_{1L}) > 0$ *and* $\frac{1}{S_2 - s_{1L} - K} + \frac{1}{K} > 0$, *then* $\partial q/\partial K < 0$.

(iii) $\partial q/\partial \sigma < 0$

Let α_{15}, α_{16}, α_{17}, α_{18} *and* α_{19}*:*

$$\alpha_{15} = [u + 1 - u(S_2 - \ln s_1 + \ln u - K)](S_2 - \ln s_1 + \ln u)$$

$$\alpha_{16} = (u + 1)(S_2 - \ln s_1 + \ln u - K)\ln\left[\frac{(S_2 - \ln s_1 + \ln u - K)}{(u + 1)K}\right]$$

$$\alpha_{17} = c_2(S_2 - \ln s_1 + \ln u)^2(S_2 - \ln s_1 + \ln u - K)u(u + 1) > 0$$

$$\alpha_{18} = (u + 1)(S_2 - \ln s_1 + \ln u)\left\{1 - \ln\left[\frac{(S_2 - \ln s_1 + \ln u - K)}{(u + 1)K}\right]\right\} < 0$$

$$\alpha_{19} = K(u + 1)\ln\left[\frac{(S_2 - \ln s_1 + \ln u - K)}{(u + 1)K}\right] - u(S_2 - \ln s_1 + \ln u - K)(S_2 - \ln s_1 + \ln u)$$

By the definition of the natural logarithm:

$$S_2 - \ln s_1 + \ln u - K > \ln(S_2 - \ln s_1 + \ln u - K).$$

By definition, $K < u(S_2 - \ln s_1 + \ln u)/(u + 1)$, *then*

$$u(S_2 - \ln s_1 + \ln u) > K(u + 1).$$

Therefore, $\alpha_{19} < 0$. *Then,* $\partial q/\partial u = (\alpha_{15} - \alpha_{16})/\alpha_{17} = (\alpha_{18} + \alpha_{19})/\alpha_{17} < 0$.

By substituting $u = e^{\sigma\Delta t}$ *and the chain rule:*

$$\frac{\partial q}{\partial \sigma} = \frac{\partial q}{\partial u}\Delta t e^{\sigma\Delta t} < 0.$$

(iv)

$$\begin{array}{ll} \partial q/\partial S_2 > 0 & , s_{1L} + \frac{K}{1 - p^*} < S_2 < \widehat{S_2} \\ \partial q/\partial S_2 = 0 & , S_2 = \widehat{S_2} \\ \partial q/\partial S_2 < 0 & , \widehat{S_2} < S_2 < s_{1H} \end{array}$$

$$\frac{\partial q}{\partial S_2} = \frac{S_2 - s_{1L} - (S_2 - s_{1L} - K)\ln[(1 - p^*)(S_2 - s_{1L} - K)/(p^*K)]}{c_2(S_2 - s_{1L})^2(S_2 - s_{1L} - K)}.$$

By definition, $c_2(S_2 - s_{1L})^2(S_2 - s_{1L} - K) > 0$.

$\Rightarrow S_2 - s_{1L} - (S_2 - s_{1L} - \widehat{K})\ln[(1 - p^*)(S_2 - s_{1L} - \widehat{K})/(p^*\widehat{K})] = 0$, *where* $\widehat{K}$ *is the critical capacity price at* $\partial q/\partial S_2 = 0$. *Then, by using simple algebra:*

$$\frac{1 - p^*}{ep^*} = \frac{\widehat{K}}{S_2 - s_{1L} - \widehat{K}} e^{\frac{\widehat{K}}{S_2 - s_{1L} - \widehat{K}}}$$

Thus, in this equation system the Lambert-W function is used to find $\widehat{K}$*:*

$$\widehat{K} = (S_2 - s_{1L})\frac{LambertW\left(\frac{1 - p^*}{ep^*}\right)}{1 + LambertW\left(\frac{1 - p^*}{ep^*}\right)}$$

Then, the critical mispricing ration, $\hat{\psi}$, *is the following:*

$$\hat{\psi} = \frac{\widehat{K}}{(1 - p^*)(S_2 - s_{1L})} = \frac{1}{(1 - p^*)}\frac{LambertW\left(\frac{1 - p^*}{ep^*}\right)}{1 + LambertW\left(\frac{1 - p^*}{ep^*}\right)}$$

Thus,

$$\begin{array}{ll} \partial q/\partial S_2 < 0 & , 0 < \psi < \hat{\psi} \\ \partial q/\partial S_2 = 0 & , \psi = \hat{\psi} \\ \partial q/\partial S_2 > 0 & , \hat{\psi} < \psi < 1 \end{array}$$

By substituting $\psi = K/(1 - p^*)(S_2 - s_{1L})$,

$$\widehat{S_2} = s_{1L} + \frac{K}{\hat{\psi}(1 - p^*)}$$

Therefore, by using $\widehat{S_2}$:

$$\begin{aligned} &\partial q/\partial S_2 > 0 \quad , s_{1L} + \frac{K}{1-p^*} < S_2 < \widehat{S_2} \\ &\partial q/\partial S_2 = 0 \quad , S_2 = \widehat{S_2} \\ &\partial q/\partial S_2 < 0 \quad , \widehat{S_2} < S_2 < s_{1H} \end{aligned}$$

(v) $$\begin{aligned} &\partial q/\partial s_1 > 0 \quad , s_{1L} < s_1 < \widehat{s_1} \\ &\partial q/\partial s_1 = 0 \quad , s_1 = \widehat{s_1} \\ &\partial q/\partial s_1 < 0 \quad , \widehat{s_1} < s_1 < \frac{1}{d} e^{S_2 - \frac{K}{(1-p^*)}} \end{aligned}$$

$$\frac{\partial q}{\partial \ln s_1} = -\frac{\partial q}{\partial S_2} \Rightarrow \frac{\partial q}{\partial s_1} = \frac{\partial q}{\partial \ln s_1}\frac{1}{s_1} = -\frac{\partial q}{\partial S_2}\frac{1}{s_1}$$

Thus,

$$\begin{aligned} &\partial q/\partial s_1 > 0 \quad , 0 < \psi < \hat{\psi} \\ &\partial q/\partial s_1 = 0 \quad , \psi = \hat{\psi} \\ &\partial q/\partial s_1 < 0 \quad , \hat{\psi} < \psi < 1 \end{aligned}$$

Since, similar to $\widehat{S_2}$,

$$\widehat{s_1} = \frac{1}{d} e^{S_2 - \frac{K}{\hat{\psi}(1-p^*)}}$$

By substituting $\widehat{s_1}$:

$$\begin{aligned} &\partial q/\partial s_1 > 0 \quad , s_{1L} < s_1 < \widehat{s_1} \\ &\partial q/\partial s_1 = 0 \quad , s_1 = \widehat{s_1} \\ &\partial q/\partial s_1 < 0 \quad , \widehat{s_1} < s_1 < \frac{1}{d} e^{S_2 - \frac{K}{(1-p^*)}} \end{aligned}$$

Proof of Proposition 11. *From Proposition 6, the first order condition gives the closed form solution for q*:*

$$q^* = \frac{1}{c_2(S_2 - s_{1L})} \ln\left[\frac{(1-p^*)(S_2 - s_{1L} - K)}{p^* K}\right].$$

Since $K > S_2 - s_{1L}$.

$$\ln\left[\frac{(1-p^*)(S_2 - s_{1L} - K)}{p^* K}\right] < \ln 1 < 0.$$

Therefore, q < 0. By the nonnegativity constraint on the capacity, q*=0.*

Proof of Proposition 12.

$$\min_q c_1\left[p e^{c_2 q K} + (1-p) e^{-c_2 q (S_2 - s_{1L} - K)}\right], subject\ to\ q \geq 0.$$

The KKT conditions are (i)$\lambda \geq 0$, (ii)$\lambda q^* = 0$and (iii) $c_1 c_2 (S_2 - s_{1L})\left[p(1-p^*) e^{c_2 q^* (1-p^*)(S_2 - s_{1L})} + (1-p) p^* e^{-c_2 p^* q^* (S_2 - s_{1L})}\right] - \lambda = 0 \Rightarrow q^* > 0$ and$\lambda = 0$. Then, $q \geq 0$is weakly active. Since $E\left[C\left(P(q, s_1)\right)\right]$ and $q \geq 0$ are convex, $q^* > 0$ is the global optimum by KKT. Hence, the first order condition gives the global minimum:

$$c_1 c_2 (S_2 - s_{1L})\left[p(1-p^*) e^{c_2 q^* (1-p^*)(S_2 - s_{1L})} + (1-p) p^* e^{-c_2 p^* q^* (S_2 - s_{1L})}\right] = 0$$

By simple algebra, q:*

$$q^* = \frac{1}{c_2(S_2 - s_{1L})} \ln\left(\frac{(1-p)p^*}{p(1-p^*)}\right) = \frac{1}{c_2(S_2 - s_{1L})} \ln\left(\frac{L^*}{L}\right)$$