



## THE VASSILIEV INVARIANT APPLIED TO FINANCIAL MARKETS

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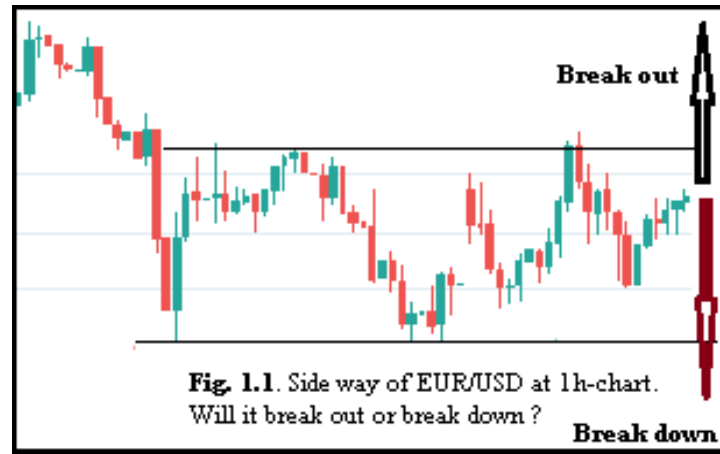
### **Abstract**

*In mathematical knot theory, a circle embedded in three-dimensional space can take many forms. The ends of a mathematical knot are joined together. As with many other concepts of higher mathematics, knot theory can be seen to have implications for price movements. A knot invariant is a quantity that is the same for equivalent knots. The concept of finite-type (Vassiliev) knot invariants involves the prolongation of knot invariants to singular knots. The singular knots have a finite number of ordinary double points. The Vassiliev prolongation is thus a type of recursion and the double point has two resolutions: positive and negative. In an earlier article [1], the author demonstrated how knot invariants can be used predictively [2,3] in price charts [4,5]. This article addresses a mechanism to make moving the market, the concepts of the phase and phase transition in the chart, proof the conjecture of Vassiliev in the financial markets[2], and the chart is a space of knots.*

**Keywords:** *Knot; Knot Invariants; Vassiliev; Price Movements; Predictive*

## SINGULARITY

When a candlestick chart is trending sideways, we do not know whether the price will break out or break down:



Knot theory can shed light on a sideways-trending chart. Consider the following three types of singularity:

### 1.a Degree\_1 Singularity

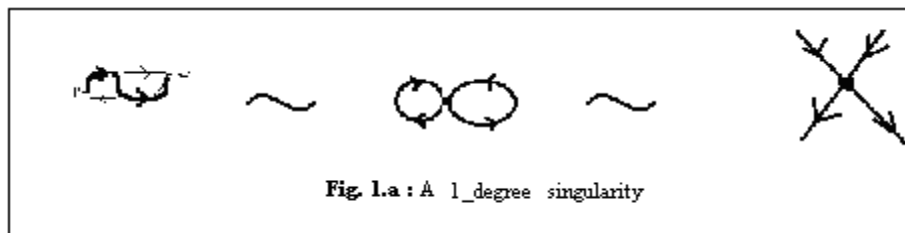
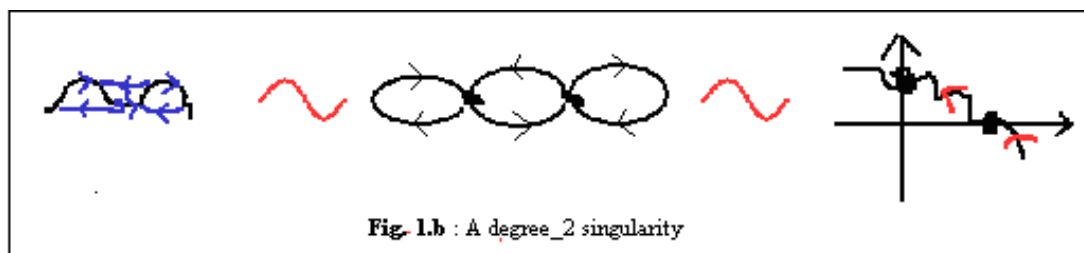


Fig. 1.a shows a 1-degree singularity in a knot equation.

Fig. 1.a.1 below shows an example of a sideways-trending chart for EUR/USD that has an identifiable 1-degree singularity (the knot is superimposed on the chart image at the relevant juncture):



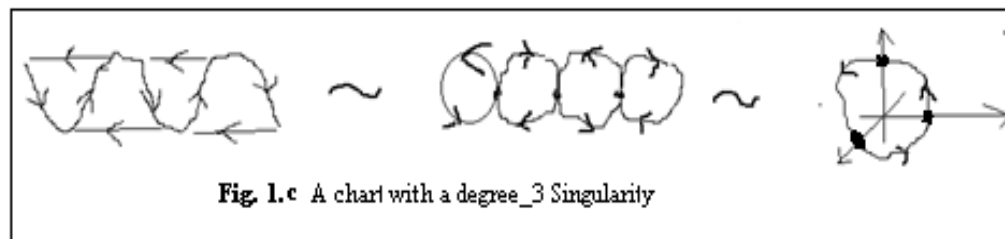
Fig. 1.b below shows a 2-degree singularity. In the simplest possible terms, the knot singularities have progressed to:



And note the point of singularity on the sideways-trending USD/CAD chart below, which has a 2-degree singularity (Fig. 1.b.1):



The important thing to remember with knot invariants and their singularities is the directionality of the knot. Look at the 3<sup>rd</sup> degree singularity below (Fig. 1.c):



The calculation of the knot invariant works like a sine wave. Below, observe the sideways-trending 1-hour EUR/USD chart for Mar 17, 2020, and the location of the 3-degree singularity (Fig. 1.c.1):



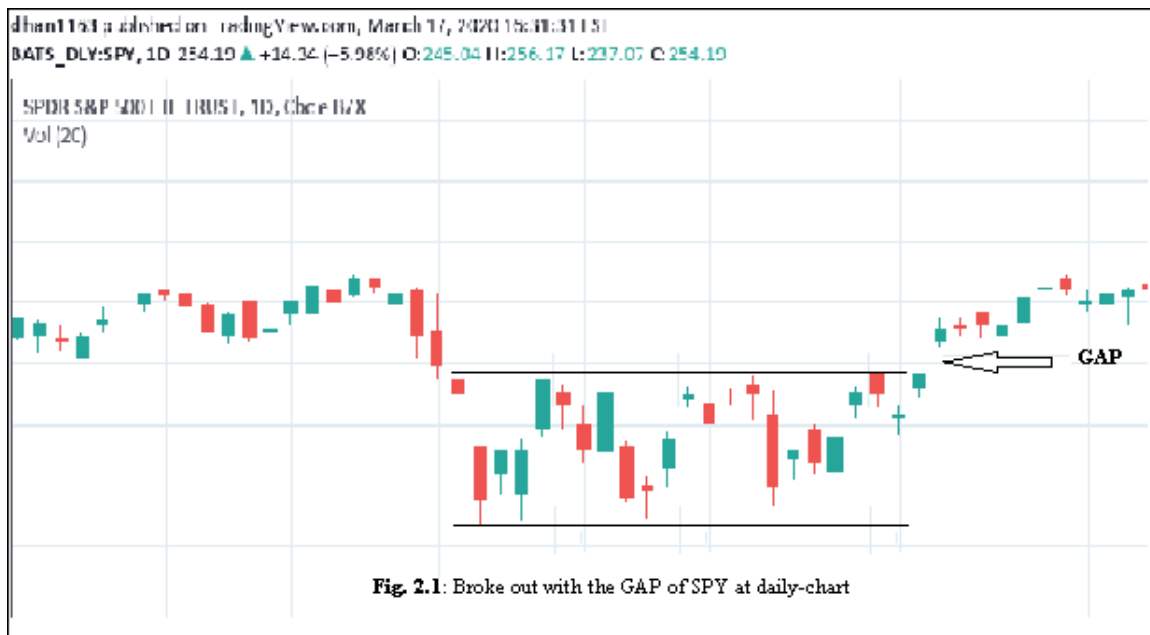
Fig.1.c.1 : Side way of EUR/USD at the 1h-chart

A breakdown is indicated.

## PHASE TRANSITION

### 2.1 Phase

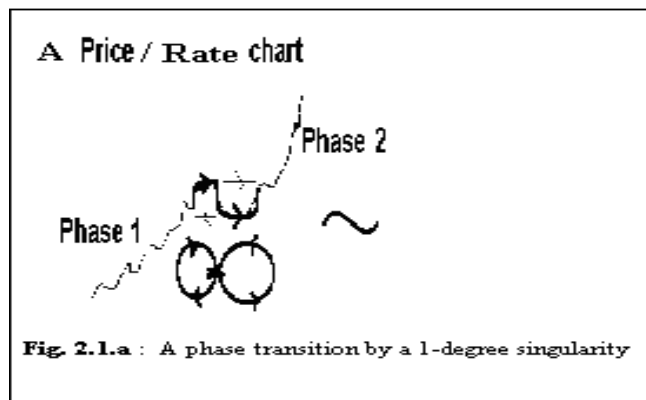
A phase of a price, by definition, is the trend of price in a certain time interval. Price is continuously moving, but phase transition is a discontinuous motion. The phase transition happens if there are singularities between two phases, creating a price spike in either direction. Hence, we have breakout or breakdown in prices. Breakout or breakdown can be attributable to news, randomness or “noise”. The price chart leaps with a gap (Fig. 2.1):



or moves vertically (Fig. 2.2):



Fig. 2.1.a below shows how a 1-degree singularity can trigger a phase 2 price trend:



In Fig. 2.1.b below, a phase transition featuring a 2-degree singularity:

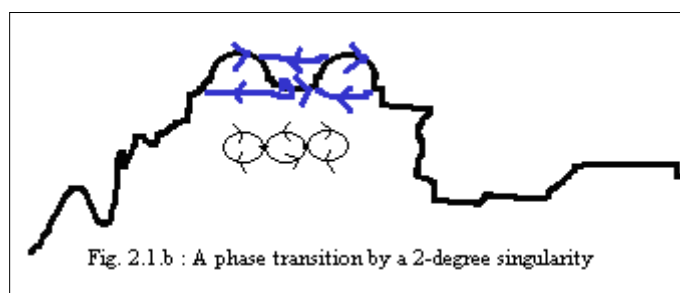
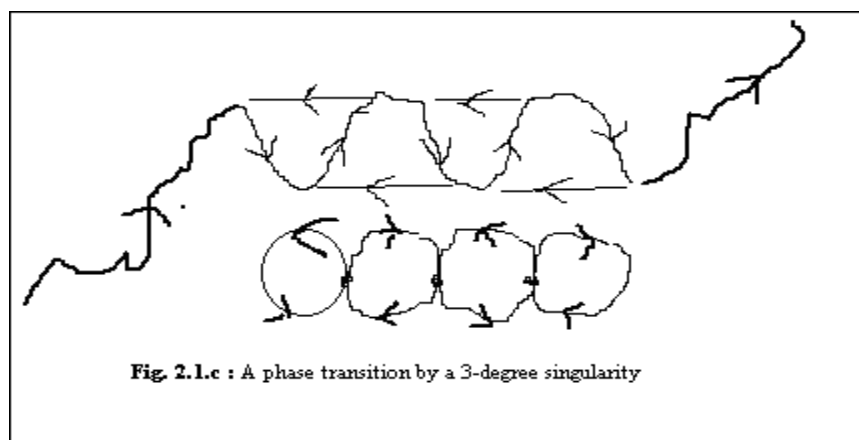


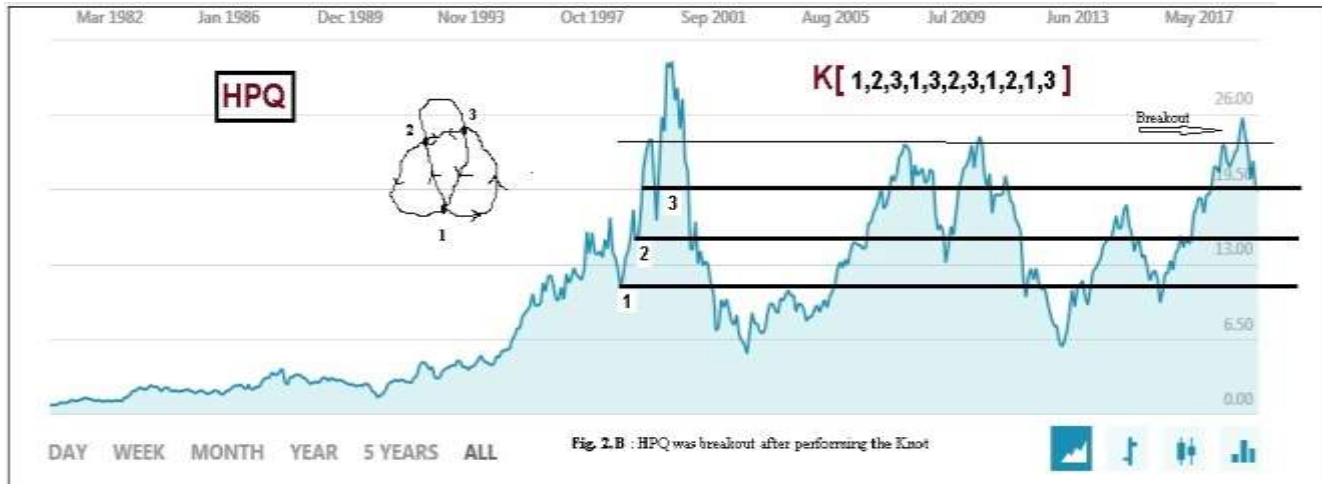
Fig. 2.1.c shows a phase transition with a 3-degree singularity:



In a knot invariant, the singularity occurs each time; the knots are equivalent, but differently-directioned.

When a security performs a knot completely, it is going to either breakout or breakdown.

The breakout of HPQ [5] after performing the trefoil knot is shown below (Fig. 2.b).



Consider the trefoil knot: “3” could go to either “2” or “1”. That is the nature of knot invariants: *the singularities are equal*. So, which way will “3” go? We don’t know. Thus, we consider “3” as a singularity. Similarly, “2” and “1” also are singularities.

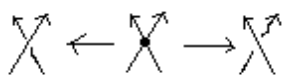
### VASSILIEV INVARIANT

A price/rate chart displays the knots. Now, we can consider the chart is a space of knots  $F$  and a function  $v: F \rightarrow \mathbb{R}$ . According to Vassiliev singular knots have a finite number of ordinary double points.

$v$  is a finite-order Vassiliev invariant. Each double-point in a singularity knot satisfies the following relation:

$$v(\text{X}) = v(\text{Y}) - v(\text{Z})$$

The Vassiliev invariant elucidates the changing directionality of the knot as it cycles:





We will denote by  $\Sigma_0$  the set of (ordinary) knots and by  $\Sigma_n$  the set of singular knots with  $n$ -double points.

Hence,

$$\mathbb{F} \supset \Sigma_0 \cup \Sigma_1 \cup \dots \cup \Sigma_n$$

Moreover, the set  $V_n$  of all Vassiliev invariants of order  $\leq n$  is a vector space and has inclusions:

$$V_0 \subset V_1 \subset \dots \subset V_n$$

We note here that a singular knot with  $n$ -double points is a knot with  $n^{\text{rd}}$  degree singularity (as we call it above).

By nature, the directionality of the knots is cyclical. Hence, we have the following normalization:

$$v(\text{positive crossing}) = v(\text{positive circle}) = 1$$

$$v(\text{negative crossing}) = v(\text{negative circle}) = -1$$

$$v(\text{a cycling knot}) = 1$$


In general,

$$|v(c)| = 1; \forall c \in \{\text{cycling knots}\} \subseteq \Sigma_0$$

A cycling knot means if we are moving either forward or backward any point on the knot, it will come back to its starting position.

**For example:**

$$|v(\text{figure-eight})| \neq 1$$

because  is not cyclical.

Moreover,

$$|v(\bigcirc \bigcirc)| = |v(\bigcirc)| + |v(\bigcirc)| = 1 = |v(\bigcirc \bigcirc)|$$

We can consider three the following examples:

**Example 1:**

$$v(K_0) = 1 ; \forall K_0 \in \Sigma_0$$

**Example 2:**

$$K_1 = \bigcirc \bigcirc$$

$$\begin{aligned} v(K_1) &= (v(\bigcirc \bigcirc) - v(\bigcirc \bigcirc)) \\ &= (v(\bigcirc) - v(\bigcirc)) \\ &= (1 - (-1)) = 2 \end{aligned}$$

**Example 3:**

$$K_2 = \bigcirc \bigcirc \bigcirc$$

$$\begin{aligned} v(K_2) &= v(\bigcirc \bigcirc \bigcirc) - v(\bigcirc \bigcirc \bigcirc) = \\ v(\bigcirc \bigcirc \bigcirc) - v(\bigcirc \bigcirc \bigcirc) - (v(\bigcirc \bigcirc \bigcirc) - v(\bigcirc \bigcirc \bigcirc)) &= \\ v(\bigcirc) - v(\bigcirc) - (v(\bigcirc) - v(\bigcirc)) &= \\ v(\bigcirc) &= 1 \text{ Because it has counterclockwise cycling property} \end{aligned}$$

Thus,  $v(K_2) = 1 - (-1) - [-1 -1] = 1 + 1 + 2 = 4$

First of all, we need to prove the following lemma:

$$\forall K_n \in \Sigma_n;$$

$$v(K_n) = 2^n$$

$v$  is an invariant  $\in \forall n; n \in \mathbb{N}$

### Proof

We can prove the lemma by mathematical induction:

With  $n = 1$ ;  $v(K_1) = v(K_1') = 2$  (see example 2),

Suppose that it is true with  $n = k$ , that is:

$$v(K_k) = 2^k$$

With  $n=k+1$

$$\begin{aligned} v(K_{k+1}) &= v\left( \underbrace{\quad\quad\quad}_{\substack{\downarrow \\ \text{k double points}}} \begin{array}{c} \nearrow \\ \searrow \end{array} \right) \\ &= v\left( \underbrace{\quad\quad\quad}_{\substack{\downarrow \\ \text{k double points}}} \begin{array}{c} \nearrow \\ \searrow \end{array} \right) - v\left( \underbrace{\quad\quad\quad}_{\substack{\downarrow \\ \text{k double points}}} \begin{array}{c} \nearrow \\ \nearrow \end{array} \right) \\ &= 2^k - (-2^k) = 2^{k+1} \end{aligned}$$

Thus,

$$v(K_{k+1}) = 2^{k+1}$$

Therefore, I have proved the conjecture of Vassiliev Invariant/finite-order invariant classification of knots. For each pair of non-equivalent knots  $K \in \Sigma_n$ ,  $K' \in \Sigma_{n'}$ ,  $n \neq n'$ ;  $n, n' \in \mathbb{N}$ , we have  $v(K) \neq v(K')$ .

## CONCLUSION

The illustrations and proofs in this article mean that we can consider any price chart as a space of knots, and use the singularities in knot theory to predict price movements. The conjecture of the Vassiliev invariant also is proven, lending weight to the presence of knot formations in price charts. Knot theory and especially finite type invariants can have significant implications for interpreting charts and predicting breakout and breakdown of price.

The space of the chart has a mechanism to make moving the market. That is the knots (unknot and trefoil knot) are created in the chart to build up a certain power to either breakout or breakdown of the chart later.

In addition, the phase transition is a discontinuous or vertical motion to make the chart keeps or reverses the trend. It is a basis for breaking symmetry of the chart.

Moreover, trading is a game of zero sum. It means that  $\Sigma (\text{profit} + \text{lost}) = 0$ , for every traders and investors.

It looks like a conservative law. Hence, the chart has a certain symmetric property. Thus, If unbalance trades  $\Sigma \neq 0$  at a some certain time (for example, traders and investors sold their holdings much more than bought at a some panic time as Covid-19 or a recession), The symmetry of the chart is broken, the market will be moving so that its chart is coming back its symmetry. This is the other mechanism that fixes an anti-symmetry to the symmetry of the chart. The mechanism also needs a certain knot (The figure-eight knot) to perform. These shall be discussed in further study as "Chaos, the financial markets, and symmetry".

## REFERENCES

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- [3] Adams, Colin C. (2001). The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots. New York: W. H. Freeman and Company.
- [4] You can see more examples of the timing in the financial markets at my blog of site <http://www.jumpthefrog.com> and my Twitter: @dhan1163.
- [5] The charts in the article are from: <https://www.tradingview.com> and <https://www.msn.com>