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OPERATIONS RESEARCH IN AGRICULTURAL AND ECONOMIC RESEARCH FOR MULTIPLE CRITERIA DECISION MAKING: A LINEAR PROGRAMMING APPROACH

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Abstract

Operations research was developed before 1940 with the objective of improving army operations through the application of mathematical techniques. Operation research in applied research is to better decision for researchers especially for economists. The operations researchers came from different subjects and branches. In other words, Operations research as a field has tried to continue with broad dedications and professional specializations approach for better solutions. Tendencies in agricultural demand for food security, the assignment for growth and sustainability, information technology and commercial power create new opportunities to support strategic investment and operations management for both productivity and profitability. To realize such potential, the agriculturalist or the community need to improve business management and healthy relationships with stakeholder, interdisciplinary coordination and the successful applications of operation research techniques mainly linear programming. Economics, agricultural economics and operations research have common and similar roots. In economics some situations are used for borderline analysis. Operations researchers explore the situations to achieve and use the actual varieties for spending the solidity through linear programming and non linear programming model results, estimate the coefficients with uncertainty and determine the constraints costs with adding or dropping the capitals or resources. This paper describes the fundamentals and applications of operational research for the improvement in agricultural researches especially during the period of increasing burden on *natural resources. In this paper authors use their skill in the field with published works, to create attention on applications of operation research techniques, and how the researcher adapt and apply operation research specially linear programming in agricultural and economic research for better decision in simple manner with appropriate understanding.*

Keywords: Agriculture, Economics, Operation Research, Linear Programming, Non Linear Programming

INTRODUCTION

Operations research techniques owe their origin to the endeavor of applications of scientific, mathematical and logical principles to the solutions of the military problems. The origin of these techniques may be traced back to the writings of F. W. Lancaster, during the First World War, to the applied mathematical analysis to army operations. Lancaster studied relationships between victory of forces and their superiority in fire power and number.

Nobel Laureate Blackett (1948) wrote two notes setting out some of the principles of operations research and the methods of analysis. Operations research groups were organized, first in Britain and then in the U.S.A., Canada and Australia. Operations research proved to be a valuable asset to the allies in bringing them victory and, therefore, the U.S. army establishments continued and supported projects for the expansion of operations research even after the battle was over.

What is Operation Research?

Operations research is a scientific approach of providing executive sections with a quantitative basis for decisions regarding the operations under their control. One of the softness of this definition is its failure to distinguish operations research from a number of other disciplines related to business problems. The description holds equally well even if users witch Operations Research' with Quality Control or Cost Accounting.

According to Yates, (1949) operational research consists of the application of technical exploration to the problems arising in organization and arrangement by methods of scientific research, mean that arrangement of observation, trial and reasoning which researchers are in the habit of using in their systematic investigations form an integral part of operation research.

Ackoff (1961) defines an operation as a set of acts required for the accomplishment of some desired outcome. He enumerates four components of an organization, viz., communication, content, control and structure. According to him, control is a matter of directing the organization toward desired objective and it is obtained by efficient decision making by those who manage the operations. Assisting managers to control organizations and improving their decision making and it is important objective of operations research. The basic important components of an operations research are:

1. Identification and formulation of the problem
2. Defining the objective function for optimization
3. Construction of a mathematical model satisfying the constraints according to the values of the variables
4. Obtaining the estimates of empirical parameters
5. Solving the model and finding out the course or courses of action that would optimize the objective function

Operation Research in Applied Research

Operation research in agricultural research is to better decision for researchers especially for agricultural economists. It is also important to knowledge with understanding about the relationship of economics and professional approach of operations research, first, need to understand some of the fundamentals of both specializations. The operations researchers came from multi-disciplinary and professional branches. In other words, Operations research as a field has tried to continue its multi-professional coordinated approach with broad objectives. Hence, operations research has played an increasingly important role in decision making in all fields of business activity such as transportation, manufacturing, purchasing and selling. Many organizations have their own division of operation researchers. Operations research techniques have been used by government and social institutions for several purposes. Operations research has established itself as one of the most important sciences in the field of business management study. It is not only helps in recognizing different situations and strategies of nature but it is also in listing another courses of action open to the decision maker and the outcomes associated with them, thus suggesting which strategy for him to choose and act under a given set of situations for the best management.

Operation Research in Agriculture

One of the first, interesting and challenging applications of operations research in agriculture was by C. W. Thornthwaite (1954) on Seabrook farm.

Yates (1949) published a paper regarding the use of operational research made in the field of agriculture in the United Kingdom. In the Second World War, Great Britain had not only to expand its food supply through increased food production, but also to economize on its imports which included things needed to increase the agricultural production.

Operations researchers have contributed significantly in economics and in agricultural economics. Samuelson (1952) accepted the importance modeling techniques and derived the association between mathematical programming and equilibrium models based on economic principles. In other words, mathematical programming became an advantageous tool for economic analysis(Samuelson 1952). GAMS modeling approach was introduced by operations researchers for the purpose of explaining general equilibrium models for evaluating national improvement strategies, to introduce the first grain storage models to protect against shortage(Gustafson 1958 and Brooke et al. 1993). William (1975) and his research team derived the microeconomic policy analysis models as Project Independence Evaluation System (PIES).

Views of Economics and Operations Research

Economics and operation research are separate fields because of the economists and operations researchers have different interests with different opinions. Economists are principally interested in qualitative economic analysis and operations researchers are more interested in assisting decision making with strong computational and professional coordination. Economists are interested in quantitative numbers and looking to measure the effect of individual assessment based decisions and econometricians are interested in computational issues and their estimation methods. Scarf (1973) developed the economic equilibrium model with algorithmic calculation and it is completed through various circumstances, scrutinizing alternative strategies, and examining the sensitivity of the parameters Scarf.

Operations researchers and computer experts have been executing inventory systems and economic researcher have been focusing on the effect of inventories in the trade sequence rather than inventory issues. The operations research tool for facts and envelopment analysis and also estimate the production functions by using an alternative technique with different suppositions and goals (Charnes et al. 1978).

Wardrop (1952) derived the equilibrium model based on times and conditions based that are directly associated to the equilibrium situations for economic longitudinal equilibrium on cost based. Beckmann et al.(1956),highlighted and established the association with economic equilibrium, the majority of the literature is available in transportation journals with an operations research approach. Nagurney (1993) described the description of different kinds of equilibrium models for different conditions.

Researchers and Modeling approach

Economic models described a lot about economic negotiators performance and operations researchers have the mathematical modeling skills to implement the economic concepts and solve the effects of economic policy and alternative strategies.

An important aspect of these kinds of policy models is that in some segments they model the decisions using optimization on behalf of the technology adoptions directly in the model. The main cause for using optimization is that the models need to have representations for policies and technologies that affect more than input and output prices and quantities and there is no past to assess the resulting decisions for some segments. Other cause include the need to link more than one sector and the existence of a convoluted information history that muddies the econometric analysis for estimating such things as a production function for utilities. The optimization models are usually simplified versions of the planning models used by the industry with coefficients based on industry aggregates. Hitchcock (1941) and Koopmans (1951), developed the first useful optimization model. Kantorovitch (1939), described linear programming models for production and distribution function including the transportation model. Stigler (1944),derived the mix model based on nutrition problems. Dantzig(1963),described the first generic linear programs with simplex algorithms to simplifying the input-output model of an economy.

The financial and commercial literatures are dominated by economic studies based on financial markets and their efficiency. Malkiel (1973) demonstrated about the stock pickers and general market. Tobin (1958) derived the decision models connected with risk and return in finance. Black-Scholes (1973) derived model based on dynamic programming for valuing options for the company. Dixit and Pindyck (1994) highlighted the important significance of dynamic programming for the undefined returns and investments.

Optimization models have a substantial role in defining the mix properties in a group, which represented the beginning of computational finance (Dixit and Pindyck 1994). Dorfman (1953) discussed the instinctive description of simple linear programming models. Charnes et al. (1952) and agricultural economics researcher Hazell (1986) develop the many linear programming model based on economic problems. Henderson (1955) and Land (1956) successfully derived the models based on coal bazaars using modeling approach due to the ability to explain large linear programs than in the past, stochastic programming models for construction portfolios have made an important mark in the field. Carino et al. (1994) for a clarification of the kind of operations research models used and recognized as “rocket scientists” in the financial press.

Averch and Johnson (1962), evaluated the Kuhn-Tucker conditions of an optimization model, somewhere a firm with maximizes earnings subject to a rate for return constraint. Usefulness is a simple concept for many circumstances when the objective can be clearly indicated as maximization of profits. However, practically we face many tradeoffs. For the conclusion, Keeney and Raiffa (1976) explained the issues related decision through multi variable usefulness. Bhatt et al. (2017) discussed the application of operations research in agriculture.

Operations researcher cannot be a genuine modeler and predictor without a noble understanding of the fundamentals economic theory. The simple algorithm technique represents the behavior of a set of independent financial progression for decisions to act on a set of inspirations in the form of expenses.

Mathematical Programming in Agriculture

The structure of a system and the objective function can be defined in terms of a mathematical models, the desired solution can be computed by means of the techniques grouped under a general heading of mathematical programming. It include tools like linear programming (integer and non-integer, price variable and resource variable, perturbation techniques, etc.) and non-linear programming (e.g., quadratic programming, concave and convex programming) and dynamic programming.

Concept of formulation of Linear Programming (Taha, 1975; Zeleny,1982& Winston,1995)**:**

Linear programming is based on objective function and constraints. In deriving of linear programming, the main objective is to find out the efficient set of extreme points of a set determined by the objective function. In the linear programming problem every extreme point is a basic feasible solution under the set of constraints. Similarly every extreme points is a basic feasible solution is also an extreme point of the set of feasible solutions.

Selection of Optimum Value:

The optimum value (Maximization) of C^t as X_j varies over j will be one or more of the extreme points of n.

Maximize $Z = C_1X_1 + C_2X_2 + \dots\dots\dots\dots + C_nX_n$

Subject to :

$a_{11}X_1 + a_{12}X_2 + \dots\dots\dots\dots\dots\dots\dots\dots + a_{1n}X_n \le b_1$

$a_{21}X_1 + a_{22}X_2 + \dots\dots\dots\dots\dots\dots\dots\dots + a_{1n}X_n \le b_2$

. . …………………… .

. . …………………… .

$a_{m1}X_1 + a_{m2}X_2 + \qquad\qquad + a_{mn}X_n \le b_n$

$X_j \ge 0$ and j= 1, 2, ……….,n

Can be written as

Max. $Z = C^t X$

Subject to

$AX \leq b$ and

$X \geq 0$

Where, X represents the vector of the variable, while C and b are vectors of known matrix of coefficient. The expression to be maximized is called the objective function C^t in this case, the equation $AX \leq b$ is the constraint which specifies a convex polyhedral set over which the objective function is to be optimized. The ($C_1, C_2, \ldots\ldots\ldots, C_n$)are the unit returns for the coming from each production process ($X_1, X_2, \ldots\ldots\ldots., X_n$).

In terms of Matrix

Max. $Z = \sum C_i X_j$ i = 1 to n

$\sum A_{ij} X_j \leq b_i$ j = 1 to n

$A_{ij} = [\, a_{ij}]_{m \times n}$

$X_j = [X_{ij}]_{m \times n}$

$b_i = [b_{ij}]_{m \times n}$

$\sum C_i X_j$ i= 1 to n

Example of Linear Programming on hypothetical data:

One of the simple but perhaps the most important and widely used techniques developed in the field of operations research is linear programming. Its applicability to the practical applications in the fields of economic management has been largely responsible for its development to the present level. The objective is to optimize (maximize or minimize as the case may be) the function f(x) where f(x) = FX + k is a vector function. It is linear.

F is a functional, k is a constant, and X ranges over a convex polyhedral set of points. The maximization or minimization of the objective function is subject to certain linear constraints.

Examples of Linear programming:

A farm produces Mangoes (x) and Oranges (y),each crop needs land, fertilizer, and time.

- 6 acres of land: $3x + y \leq 6$
- 6 tons of fertilizer: $2x + 3y \leq 6$
- 8 hour work day: $x + 5y \leq 8$
- Mangoes sell for twice as much as Oranges
- Maximize profit (z): $2x + y = z$
- We can't produce negative: $x \geq 0$, $y \geq 0$

Examples of Nonlinear programming model:

The fundamental assumption of linear programming is that all objective and constraint functions are linear. Although this assumption essentially holds for numerous practical problems, there are many problems which doesn't satisfies the assumptions Practical optimization problems frequently involves nonlinear functions.

Maximize $U=5x + y - (x+y)^2$

Subject to

$x+y \leq 2$

and $x \geq 0, y \geq 0$.

Is an example of nonlinear model which can be answered by the simple alteration of the simplex method resulting $x=3/2, y=1/2$ and $U=3$.

How to solve the problems?

Researcher can solve the problem with the help of simple graphical methods and simplex method or by using any software (Microsoft Office Excel, Lingo, GAMS, etc.) During the applications of any these software understanding is important about the formulation of problems, decision variable, Objective function, constraints etc.

Decision Variable, Objective Function and Constraints in hypothetical example:

Concepts of decision variable, objective function, constraints and formulation of linear programming problem based on given simple problem.

Example: A manufacturer is making cups and plates. It takes her 6 minutes to make a cup and 3 minutes to make a plate. Each cup uses 3/4 kg. of clay and each plate uses one kg. of clay. He has 20 hours available for making the cups and plates and has 250 kgs. of clay on hand. Manufacturer makes aprofit of E 8.00 on each cup and E 6.00 on each plate. How many cups and how many plates should he make in order to maximize his profit?

Decision Variables: In the linear programming problem, the decision variables should completely define the decisions to be made.

X = Number of produce (Cup)

Y = Number of produce (Plate)

Objective Function: The objective function characterizes the objective that manufacturer is trying to attain. In the case is to maximize (max) total contribution margin. For each cup that is sold, E 8 in contribution margin will be realized. For each plate that is sold, E 6 in contribution margin will be realized. Thus, the total contribution margin for manufacturer can be expressed by the following objective function equation:

$$\text{Max. } Z = 8X + 6Y$$

Where; Z denotes the objective function value of Linear Programming Problem.

Constraints: A constraint is basically some control under which the manufacturer necessity for work, such as limited production time or raw materials. In this case, the objective function grows larger as X and Y increase. In other words, if manufacturer were free to choose any values for X and Y, the manufacturer could make an arbitrarily large involvement margin by selecting X and Y to be very large. The values of X and Y, however, are controlled by the following two constraints:

Constraint 1: Each day, no more than 20 hours (1200 minutes) of manufacturing time may be used. Thus, constraint l may be expressed by:

$$6X + 3Y \leq 1{,}200$$

because it takes 6 minutes of manufacturing to produce one product (X) and 3 minutes of manufacturing to produce another one (Y). The inequality sign means that the total soldering time for both products X and Y cannot exceed the 1,200 manufacturing minutesavailable, but could he less than the available minutes.

Constraint 2: Produce (X) and Produce (Y) need clay but not more than 250 kgs. (250000gms.)Thus, constraint 2 may be expressed by:

$$750X + 1000\,Y \leq 250000$$

Sign of Restrictions: To complete the formulation of a linear programming problem, the following question must be answered for each decision variable. Decision variable only assume only nonnegative values, or is it allowed to assume both positive. In most linear programming problems, positive values are assumed. In this case, production cannot be less than zero units. Therefore, the sign restrictions are:

X >= 0 and Y >= 0

Solve the Hypothetical problem by using the Simplex method:

Maximize	$Z = f(x,y) = 40x + 50y$
subject to:	$4x + 3y \leq 120$
	$x + 2y \leq 40$
	$x \geq 0$, $y \geq 0$

Consider the following simple steps:

STEP-1: Make a change of variables and normalize the sign of the independent terms.

A change is made to the variable naming and establishing the following: x becomes X1 and y becomes X2

As the independent terms of all restrictions are positive no further action is required. Otherwise there would be multiplied by "-1" on both sides of the inequality (noting that this operation also affects the type of restriction).

STEP-2: Normalization of restrictions.

The inequalities become equations by adding slack, surplus and artificial variables as the following table:

Inequality type	Variable that appears
≥	- surplus + artificial
=	+ artificial
≤	+ slack

In this case, a slack variable (S1 and S2) is introduced in each of the restrictions of ≤ type, to convert them into equalities, resulting the system of linear equations:

$$4X1 + 3X2 + S1 = 120$$

$$X1 + 2X2 + S2 = 40$$

STEP-3: Equate the objective function to zero.

$Z - 40X1 - 50X2 - 0 \cdot S1 - 0 \cdot S2 - 0 \cdot S3 = 0$

STEP-4: Write the initial tableau of Simplex method.

The initial tableau of Simplex method consists of all the coefficients of the decision variables of the original problem and the slack, surplus and artificial variables added in second step.

STEP-5: Stopping condition

If the objective is to maximize, when in the last row there is no negative value between discounted costs the stop condition is reached.

In that case, the algorithm reaches the end as there is no improvement possibility. The Z value is the optimal solution of the problem.

Another possibility if all values are negative or zero in the input column variables of the base. This indicates that the problem is not limited and the solution will always be improved.

Otherwise, the following steps are executed iteratively.

STEP-6: Choice of the input and output base variables.

First, input base variable is determined. For this, column whose value in Z row is the lesser of all the negatives is chosen.

If there are two or more equal coefficients satisfying the above condition, then choice of the basic variables.

The column of the input base variable is called *pivot column*.

Once obtained the input base variable, the output base variable is determined. The decision is based on a simple calculation: divide each independent term between the corresponding values in the pivot column, if both values are strictly positive. The row whose result is minimum score is chosen.

If there is any value less than or equal to zero, this quotient will not be performed. If all values of the pivot column satisfy this condition, the stop condition will be reached and the problem has an unbounded solution.

STEP-7: Update tableau.

The new coefficients of the tableau are calculated as follows:

In the pivot row each new value is calculated as:

New value = Previous value / Pivot

In the other rows each new value is calculated as:

*New value = Previous value - (Previous value in pivot column * New value in pivot row)*

Solution of the Above Problems (Tableau):

1st iteration							
Basic Variable	Z	X1	X2	S1	S2	RHS	Ratio
Z	1	-40	-50	0	0	0	
S1	0	4	3	1	0	120	40
S2	0	1	2	0	1	40	20
2nd iteration							
Basic Variable	Z	X1	X2	S1	S2	RHS	Ratio
Z	1	-15	0	25	0	1000	
S1	0	5/2	0	-3/2	1	60	24
X2	0	1/2	1	1/2	0	20	20
3rd iteration							
Basic Variable	Z	X1	X2	S1	S2	RHS	Ratio
Z	1	0	0	16	6	1360	
X1	0	1	0	-3/5	2/5	8	
X2	0	0	1	4/2	-2/10	24	

Z= 1360 X=8 and Y = 24

Max. $Z_{(8, 24)} = 1360$

Applications of linear programming in agriculture

In agriculture linear programming has been used in agriculture almost since its very beginning. In 1951 Waugh applied this technique to the problem of minimization of costs of feed for dairy problems. Koopmans (1951) derived activity analysis of production and resource use. Hildreth and Reiter (1951)had a paper entitled, On the Choice of a Crop Rotation Plan. Heady and Candler (1958) used linear programming methods, deals exclusively with applications in the field of agriculture. Boles (1955) had published article on "Linear Programming and Farm Management Analysis. Perhaps the most extensive use of linear programming in agriculture has been in the field of feed-mixing with the object of minimization of cost of feed. Use of linear programming for individual farmers is commonly referred to as program planning and has been widely used in Europe and Japan and to a limited extent in the USA. Barker (1964)conducted a study on the use of linear programming in making farm management decisions and came to the conclusion that, linear programming can be of value in farmer decision making by providing quantitative estimates of returns for specified alternatives and levels of resource use and the larger the size of the farm, the larger the number of alternatives and the greater the likelihood of benefits from linear programming exceeding its costs. In addition to their use at the micro level, i.e., cost minimization and profit maximization on an individual farm, linear programming techniques have been applied with advantage at the macro level for solving the problems of agricultural marketing and spatial analysis. Studies in inter regional production and adjustments for major crops have been made through the use of spatial linear programming technique. Transportation models, assignment models and simple linear programming models also used in applied agricultural research.

SUMMARY AND CONCLUSIVE REMARKS

Economist and operations researcher have common interest. In economics conditions are used for marginal analysis, as with the search for institutional alterations of the marketplace and for such uses as the derivation of cost functions from production functions. Operations researchers exploit these conditions to expand algorithms and use the real choices for assessing the stability of the model results, estimating the effects of uncertainty in the coefficients and defining the expenses of constraints with an adding or dropping resources.

Applications of these techniques reflect the different professional goals of the researcher engaged in these fields. Agriculturalists always thinking about lands, seeds, fertilizers, irrigation and their combination for better productivity, transportation, market etc..Operations researchers' emphasis on making better decisions and economists study the consequences of different market structures and strategies through an assumption of sensible decision. These groups are interested in better management, understanding sensible decision making and the consequences of balanced decisions. This can be realized in the different views and opinions of the firm. In the traditional agriculture the farmers always on risk and dependent on others issues. The old-style economic theory of the firm is really a theory of the interactions within the company. However, operations research models provide a theory of decision making within the firm and an important component theory. The operations research models do not provide a complete theory of decision making in the firm because operations researchers, although commenting on conflicts in the firm, tend to not focus on the incentives and structures that create these conflicts. These fields come together when there is the need to improve the decision according to the needs and create opportunities to better productivity and provide a profit as in policy analysis and finance. This paper is useful for young agricultural economic researchers, managers and also for policymakers.

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