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CAPITALIZATION (PREVENTION) OF PAYMENT PAYMENTS WITH PERIOD OF DIFFERENT MATURITY FROM THE PERIOD OF PAYMENTS

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Abstract

The cash maturity practice at the moment of payment is applied in all contracts that banking institutions associate with their depositing clients or borrowers. But this does not exclude the review of annuity cases where the maturity of payments becomes more frequent or less frequent than the payment of payments. In the dynamics of daily life, there are also annuities with a maturity period different from the payment period. For illustration, let's present the following situation: Let's assume that a person wants to accumulate in his bank account a fund to be available during retirement years. In fulfillment of his wish he deposits every 3 month end the amount of 500 € with an interest rate of 5%. Its deposits continue periodically for a certain period of time. Subsequently, the accumulated fund deposits it with the same interest rate until the moment of retirement. After retirement she withdraws every month the sum of 100 €. Let's assume, however, that after making a certain number of withdrawals, the person wants to know the situation in his bank account. To fix this situation, actions should be made for two financial operations: the deposit operation and the withdrawal operation. The deposit series forms a deposited annuity, the future value of which is matched by the formula: $S_n = R \cdot \frac{(1+i)^n - 1}{i} \cdot (1+i)^u$ *Where, (Sn) amount of an ordinary annuity of (n) payments, (R) periodic rent is the size of each payment which are made at the end of each period, (i) the* *interest rate for a maturity period, (n) the number of conversion periods and (μ) the number of maturity periods after the last payment made. The withdrawal series forms another annuity with different specifications from the annuity formed by deposits. Under the agreement drawn up at the initial point (the agreement remains in force for all time) the money matures at every 3 month end and is withdrawn each end month. As we can see, during the second financial operation we have to do with an annuity where the maturity period is different from the payment period.*

Key words: Maturity, Future Value, Present Value, Payment period, Annuity, Interest Rate, Capitalization, Compound Interest, Effective Annual Rates

INTRODUCTION

For rents with the same maturity as the payment period, the applicable formulas are:

$S_n = R\frac{(1+i)^n - 1}{i}$ (to calculate the Future Value) and $A_n = R\frac{1-(1+i)^{-n}}{i}$ (to calculate the Present Value). We note with (m) the number of maturity per year and (p) the number of payments per year ($m \neq p$). The interest rate for a maturity period is equalized by the formula:

$i = \frac{r}{m}$ Where, (r) is the annual interest rate. The fraction $k = \frac{m}{p}$ indicates the number of maturity periods that contains a period of payment.

Annuity with maturity different from the payment period

Annuities with maturity periods different from the payment period are of two types:

- Annuities with more frequent maturities than payments;
- Annuities with less frequent maturities than payments.

Figure 1 shows a more frequent maturity than the payments. A payment period contains four maturity terms.

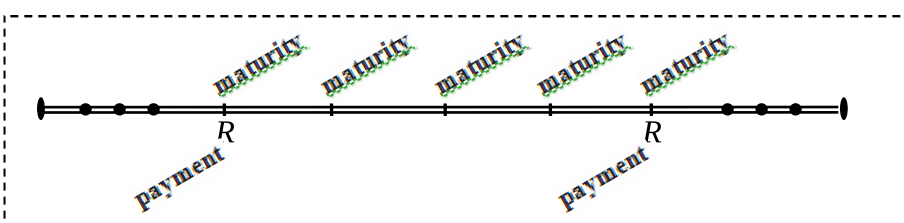

Figure 1 More frequent maturity than the payments

Figure 2 shows an annuity with less frequent maturities than payments. In this figure, a maturity period contains four payment periods.

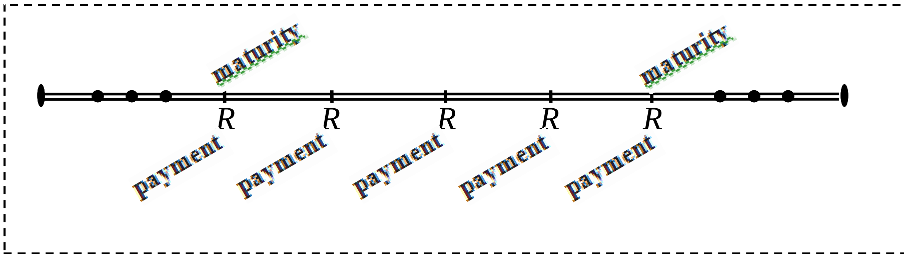

Figure 2 Annuity with less frequent maturities than payments

In Figure 1 and Figure 2 payments are made at each end of the payment period. In these types of annuities formulas for future value or present value of ordinary annuity, cannot be applied directly. In these formulas there is interest rate (*i*). According to this interest rate, the maturity of the money is made at the time of payment. Therefore, the interest rate (*i*) is used in any case when the maturity of the money coincides with the moment of payment and cannot be used in cases when the maturity date does not match with the moment of payment.

We note with:

p: Number of payments made per year;

j: interest rate for a period of payment;

k: the number of maturity periods that contains a period of payment.

Meanwhile, according to symbolism used for capitalization of simple or compound interest, for capitalization or actualization of annuities the symbol (*m*) shows the number of maturity periods in a year and the symbol (*i*) indicates the interest rate for a maturity period.

The number (*k*) is equalized by equation, $k = \frac{m}{p}$

Formulas on Capitalization (Actualization) Of Annuity

Since two ordinary annuity formulas become applicable also for annuities with maturity periods different from the interest rate period (i), the interest rate (i) should be replaced by the rate (j) for a period of payment.

The interest rate (*j*) should be such that it meets two conditions:

- mature the money at the moment of payment; - be equivalent[1] to the rate (*i*).

[1] *Two interest rates are equivalent to each other if they generate the same future value over the same period and for the same principal.*

To derive a formula that sets the interest rate (j) we base on the equation of their effective[2] annual rates. The effective annual rate of interest rate (i) is equalized by the formula:

i_{ef} (maturity) = $(1 + i)^m - 1$

The effective annual rate of interest rate (j) is equalized by the formula:

j_{ef} (of payments) = $(1 + j)^p - 1$

By equating these two effective annual rates we have:

j_{ef} (of payments) = i_{ef} (matured)

$(1 + j)^p - 1 = (1 + i)^m - 1$

$(1 + j)^p = (1 + i)^m$

$$1 + j = (1 + i)^{\frac{m}{p}}$$

$$\boxed{j = (1+i)^k - 1} \qquad \textbf{(1)}$$

The interest rate (j) has the specificity that achieves the matching of the payment period with the maturity period. According to the interest rate (j) each payment period is simultaneously a maturity period. Thus, the total number of maturity periods is now equal to the total number of payments.

Well, $\boxed{n = p \cdot t}$ **(2)**

Using the reconciled interest rate (j) by equation (1) and the number of maturity periods (n) equalized by the equation (2) formulas for the future value or the present value of the annuity commonly converted into valid annuity formulas with maturity dates different from the payment period.

For an annuity with maturity periods different from the payment periods

The future value is equated by formula:

$$\boxed{S_n = R \cdot \frac{(1+j)^n - 1}{j}} \qquad (\textbf{3})$$

[2] *The effective annual rate (i_{ef}) is the annual compound interest rate equivalent to the annual interest rate (r) maturing several times a year*

Let us refer to Figure 3:

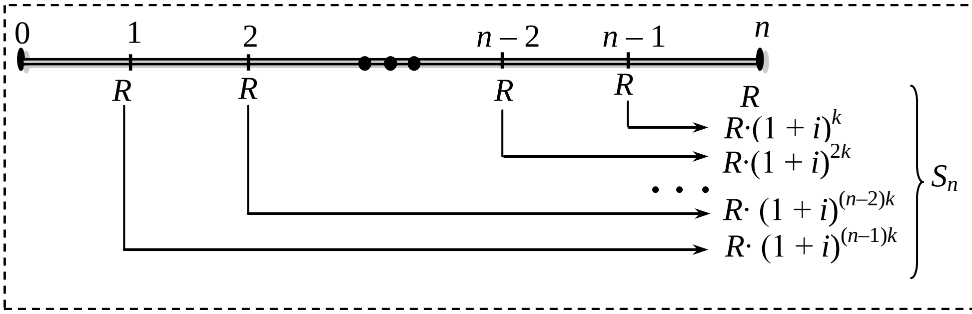

Figure 3 The future value

The following table shows the future value of each payment as well as the number of periods during the payment generates interest.

Table 1 Future value of each payment and number of periods during the payment generates interest

Payment:	Payment generates interest for:	Future Value
Last one	0 (zero) maturity periods	R
The penultimate	k maturity periods (1 payment period)	$R\cdot(1 + i)^{k}$
Third from the end	$2k$ maturity periods (2 payment period)	$R\cdot(1 + i)^{2k}$
…	…	…
Second	$(n – 2)k$ maturities ($n – 2$ payment period)	$R\cdot(1 + i)^{(n-2)k}$
First	$(n – 1)k$ maturities ($n – 1$ payment period)	$R\cdot(1 + i)^{(n-1)k}$

Starting with the last payment, the sum of the future values of all payments with R value (the future value of the annuity with maturity dates different from the payment period) is:
$S_n = R + R(1 + i)^k + R(1 + i)^{2k} + \ldots + R(1 + i)^{(n-2)k} + R(1 + i)^{(n-1)k}$.

If we write $q = (1 + i)^k$ this equation takes the form:
$S_n = R + Rq + Rq^2 + \ldots + Rq^{n-2} + Rq^{n-1}$.

The right side of this sum forms a geometric progression with the first limit (R), the quotient (q), and the number of the sums (n). We have:

$$S_n = R\cdot\frac{q^n - 1}{q - 1} = R\cdot\frac{(1 + q - 1)^n - 1}{q - 1}$$

If we write $j = q - 1 = (1 + i)^k - 1$ this equation takes the form:

$$S_n = R \cdot \frac{(1+j)^n - 1}{j}.$$

In the same way, the formula for calculating the present value is:

$$\boxed{A_n = R \cdot \frac{1-(1+j)^{-n}}{j}} \qquad \textbf{(4)}$$

RECOMMENDATIONS

1. To calculate the future value (S_n) or the present value (A_n) of a rent with more frequent maturity than the payment, we follow two steps:

First, the interest rate (j) is calculated for a period of payment equivalent to the rate (i) for a maturity period by applying the formula (2).

Second, depending on the demand expressed in the problem situation, is applied formula (1) if the future value (S_n) is applied or formula (3) is applied if the present value (A_n) is required.

2. The procedure for calculating the actual payment for a more frequent maturity than the payment is: *First,* the interest rate (j) is calculated for a period of payment equivalent to the rate (i) for a maturity period by applying the formula (2). *Second,* apply the formula (1) if the future value (S_n) is applied or formula (3) is applied if the present value (A_n) is given.

3. To calculate the future value (S_n), the present value (A_n) or the periodic payment (R) of a rent with less maturity than the payments and payments between the two maturity dates is done with compound interest rate formulas of the settlement of the problem situation is the same as the procedure applied to rents with more frequent mature payables than payments.

4. To calculate the future value (S_n) or the present value (A_n) of a rent with less frequent maturity than the payments and payments between the two maturity dates do not generate interest or generate interest under simple interest formulas, follow as: *First,* the value of virtual payment (R_v) is calculated as the arithmetical amount of real payments between maturity dates if these payments do not generate or how many of the matured amounts if the payments between the two maturity dates generate interest under the simple interest formulas. *Secondly,* depending on the demand expressed in the problem situation, apply formula (4) if the future value (S_n) is applied or formula (5) is applied if present value (A_n) is required.

5. For the calculation of the periodic payment (*R*), when the values of the other parameters of the rent with less maturities than the payment are given, and the payments between the two maturity dates are capitalized with simple interest formulas, apply the formula (4) if it is given (S_n) or (5) if it is given (A_n). By applying one of the formulas we calculate the virtual payments (R_V). Equating the virtual payment calculated with the matured amount (the arithmetical amount when the payments between the two maturity dates do not generate interest) of the real payments is formed a first-rate equation with an unknown whose solution gives the value of the real payment.

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