

# **REDUCTION OF EXPENDITURE ON THE PROVISION OF PRODUCTION AND TESTING OF ARTICLES PRODUCED IN LIMITED BATCHES**

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## **Abstract**

*In the production of single batches of expensive and complex specialized articles with limited quantities, often arises the problem to determine the quantity of elements and components to be ordered. In addition, especially with expensive articles and those whose tests are related to their destruction, occurs the problem of conducting the tests for determination of their reliability characteristics. The article proposes an approach for calculation of the number of additional components that have to be ordered for the production of a limited volume batch, as well as for the determination of the number of test pieces of the products batch. The main purpose is to be proposed a method for determining the amount of additional resources needed to guarantee a successful production of a limited series with a certain probability. The scope of the study is in the area of providing additional production resources and additional testing products.*

*Keywords: minimal losses, expenditures, production, limited batches, production resources, testing products, additional components*

## **INTRODUCTION**

In the recent years, the development of technologies and the irregularity of the orders for specialized sensors and other systems (products), required their production to be carried out occasionally, in limited series or batches and with the need to replace part of their details (Stoichev K., 2010).

The latter is due either to the lack of these details with which the articles have been assembled in their previous production or to the change of the appearance with a more modern look or the

use of newer and inexpensive substitutes. This situation poses two important issues related to the planning of supplies of the relevant components and the testing and assessment of the reliability of the systems.

The first problem is that due to factory defects and manufacturing wastage, for the accomplishment of a limited amount order is required to be claimed a certain amount of additional components, which to provide with a certain probability (but without unjustified costs) the successful production of the items planned for delivery. If the extra quantity is too great, the effect will be related to unjustified costs. If this quantity is insufficient, there is a risk that the production will be delayed until the supply of new additional components and details.

Let us assume that for the production on a limited amount batch, are needed "K" number of components (elements) of a certain type. Let it also be known that the probability for absence of a defect of this component (due to a defective component delivered or a defect in the course of the manufacturing of the article) is "p".

In this case, arises the question of determining the quantity of the components to be delivered of the considered type ( $K_d$ ), which ensures with minimal losses, the production of articles requiring "K" number of regular components. Criteria for losses in the production of limited volume batches should use the financial losses and in particular, the minimum financial loss criterion.

Let us assume that the value of one of the delivered components of the product is " $\alpha$ " financial units, the value of the penalties for late deliveries is " $\beta$ " financial units per unit of time, and the time for delivery is "t". In this case, if " $K_d$ " components were delivered, and "X" of them proved to be regular, two types of financial losses are possible.

The first type of possible loss ( $Z_{1x}$ ) is related to the supply of more than the required regular components, ie. it is a situation where  $X > K$ . Obviously this loss will be defined as:

$$(1) Z_{1x} = \alpha(X - K)$$

The second type of possible loss ( $Z_2$ ) appears in case that the regular components have been found to be less than the required number (K), therefore it takes time to provide the additional amount of components. As a result, the production and delivery of the batch is delayed, which is associated with financial losses for penalties.

For batches with limited volume it can be assumed that they are delivered to the buyer in full (not in individual tranches) and the additional supply of the missing components does not depend on their quantity. Plus, in most cases, the losses  $Z_2$  are a percentage ( $\beta_0$ ) of the total value of the contract (T) per unit of time (t), as the penalty can not exceed a certain maximum value ( $\beta_m$ ), which is reached after the delay in delivery exceeds a certain maximum period ( $t_m$ ). And if the delivery exceeds some critical time ( $T_k$ ), most contracts provide for their termination,

and the manufacturer owes penalty fees to the consumer which represent a certain percentage ( $\beta_k$ ) of the value of the contract. If the time delay of delivery exceeds a critical time ( $T_k$ ) in most contracts there is a clause for their termination and the manufacturer has to pay penalties to the consumer, representing a certain percentage ( $\beta_k$ , mostly  $\beta_k \geq 100$ ) of the contract value.

In this case, the second type of loss will only occur if the regular components have at least one piece less than the number of components required to produce the batch "K", and will have the following dependence:

$$(2) Z_2 = \beta_0 t \text{ at } t < T; Z_2 = \beta_m \text{ at } T_k > t \geq T; Z_2 = \beta_k T \text{ at } t \geq T_k$$

In most cases it can be assumed that supply planning, production and the completion of the contract, ensure fulfillment of the condition  $t < T$ , which is why the material will review this typical situation.

The probability ( $P_{1x}$ ) for the occurrence of the loss  $Z_{1x}$  can be determined by the binomial law, expressing the probability that, in the case of the supplied "Kd" components, there will be exactly "X" number of regular components (Nachev A. 2014), ie.:

$$(3) P_{1x} = C_{Kd}^X p^X (1-p)^{(Kd-X)}$$

where  $C_{Kd}^X = Kd! / (X!(Kd-X)!) -$  binomial coefficient.

Obviously the average amount of this type of loss ( $Z_1$ ) can be calculated as (Nachev A. 2014)

$$(4) Z_{1cp} = \sum P_{1x} Z_{1x} = \sum \alpha (X-K) p^X (1-p)^{(Kd-X)} Kd! / (X!(Kd-X)!)$$

where the summation is performed for values of "X" from (K+1) to Kd.

The probability ( $P_2$ ) for the occurrence of loss  $Z_2$  is the probability that of "Kd" number of components supplied, the faulty ones will be at least (Kd-K+1). Obviously  $P_2$  can be determined using binomial law [2] and replacing "p" with "(1-p)", ie :

$$(5) P_2 = C_{Kd}^X p^{(Kd-X)} (1-p)^X$$

The average value of this type of loss  $Z_{2cp}$  can be calculated by the expression (Nachev A. 2014, Kobzarev A.I):

$$(6) Z_{2cp} = \sum Z_2 P_2 = \sum \beta t C_{Kd}^X p^{(Kd-X)} (1-p)^X$$

Where summation is performed for values of "X" from (Kd-K+1) to "Kd"

The total average of the financial losses ( $Z_{2cp}$ ) in the delivery of "Kd" components for the case under consideration, can be determined by the expression [3]:

$$(7) Z_{2cp} = Z_{1cp} + Z_{2cp} = \sum \alpha (X-K) p^X (1-p)^{(Kd-X)} Kd! / (X!(Kd-X)!) + \sum \beta t p^{(Kd-X)} (1-p)^X Kd! / (X!(Kd-X)!)$$

where the first summation is performed for values of "X" from (K+1) to "Kd", and the second for "X" from (Kd-K+1) to "Kd"

Apparently with the increase of "Kd" at fixed values for the other components, the first Formula 7 collectable will increase and the second collectable will decrease (because it will increase the

probability of loss due to excessive number of components and will reduce the losses for any additional supply of missing regular components).

The optimal value of  $K_d$  ( $K_{do}$ ) is the one that in the given situation provides a minimum average losses ( $Z_{2cp}$ ). A limited quantity of the batch (and hence of "K") can be assumed that the amount of " $K_{do}$ " exceeds with a limited number the required quantity of the regular "K" components. In this case, " $K_{do}$ " can be determined by a numerical path, in which successive rising values of " $K_d$ " are set in Formula 7 until reaching a minimum value of  $Z_{2cp}$ . For this purpose, it is convenient to introduce the coefficient  $\lambda$  as the ratio between the cost of one component  $\alpha$  and the delay penalty per unit of time  $\beta t$ , ie.:

$$(8) \quad \lambda = \alpha / \beta t$$

In this case, the minimum amount of the average financial risk, for delivery of " $K_d$ " components, will be equal to the minimum of the function  $Z_{cp}$ , where:

$$(9) \quad Z_{cp} = \lambda \sum (X - K) p^X (1 - p)^{(K_d - X)} K_d! / (X!(K_d - X)!) + \sum p^{(K_d - X)} (1 - p)^X K_d! / (X!(K_d - X)!)$$

where the first summation is performed for values of  $X$  from  $(K+1)$  to  $K_d$ , and the second - for  $X$  from  $(K_d - K + 1)$  to  $K_d$ .

The dependency of  $Z_{cp}$  from " $K_d$ " for some values of  $\lambda$  and  $p$  for  $K=10$  are shown in Figures 1 and 2.

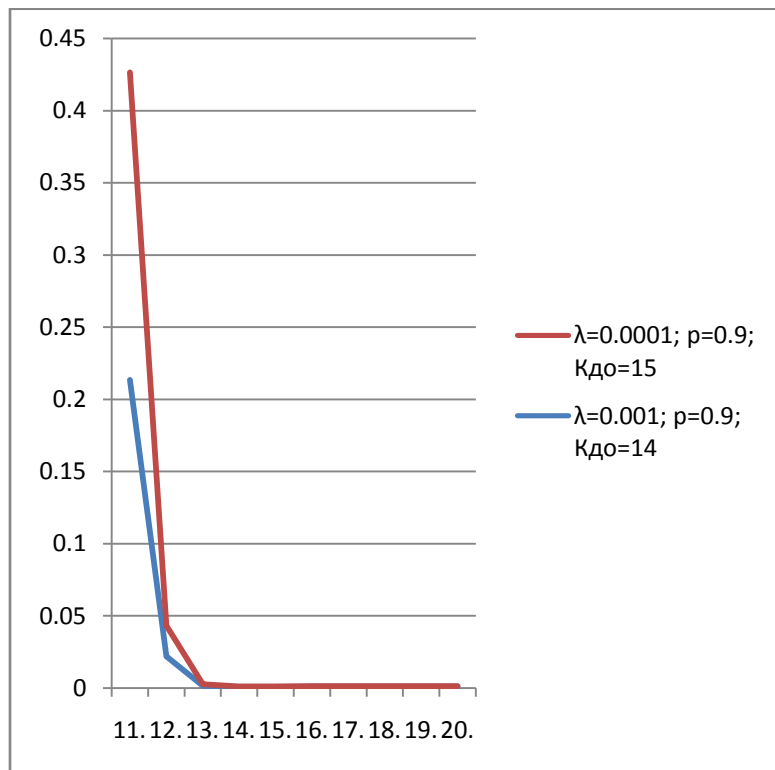


Figure 1 Dependency of  $Z_{cp}$  from  $K_d$  when  $p=0.9$

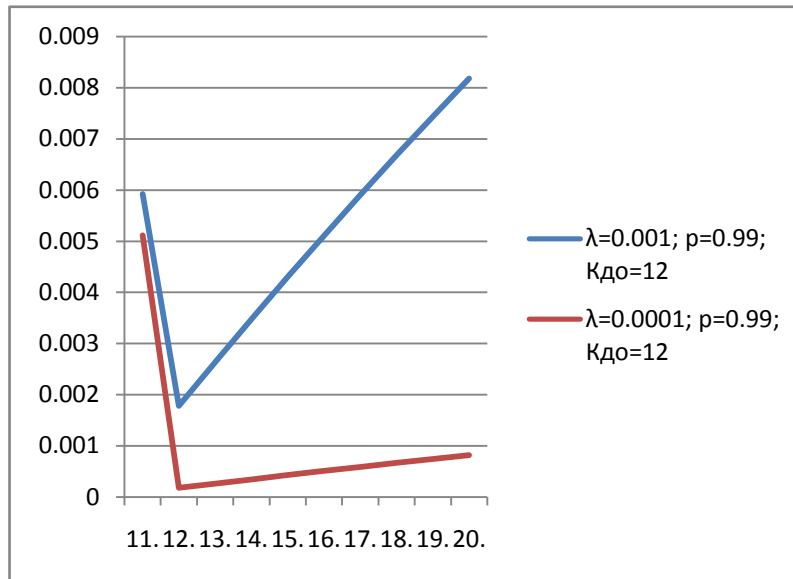


Figure 2 Dependency of  $Z_{cp}$  from  $K_d$  when  $p=0.99$

It can be seen from Figure 1 that with the increase of  $\lambda$  one order does not lead to a significant increase in  $K_{dO}$ , while when the probability for the reliability of the delivered components is high ( $p = 0.99$ ) the deviation of  $K_{dO}$  leads to a sharp increase in the average financial risk, and specially at higher  $\lambda$  values. Additionally, the graphical dependencies of Figures 1 and 2, as well as other calculations made by the said methodology, show that the required amount of additional components varies within 20-30% of the amount needed to make the batch - ie.  $K_{dO}=(1.2-1.3)K$ .

The second major problem in the manufacture of specialized articles in limited quantities with replaced details, relates to the tests necessary to determine their reliability characteristics. Because of the limited batch volumes, because of their high price per unit, and due to the considerable cost of financial resources and test time, it is necessary to plan the quantity of the series of articles to be tested. Naturally the most economical method for the planning of the tests in this case is to conduct them until the first failure occurs (Nachev A. 2014, Kobzarev A.I). It is then necessary to specify a test series containing "N" number of articles, which to ensure a high probability of occurrence of the first failure.

Assuming that "p" is the probability for a product from the batch to be without defects, then according to the geometric distribution, the probability of occurrence of the first failure ( $P_N$ ) can be defined as (Nachev A. 2014)

$$(10) \quad P_N = 1 - p^N$$

From where, at a set probability value  $P_N$ , for the quantity of items in the series to be tested we get:

$$(11) \quad N = \ln(1 - P_N) / \ln(p)$$

Figure 3 shows the dependencies of "N" from  $P_N$  for some typical values of "p". It can be seen that if small values are acceptable for  $P_N$  (around 0.7), then about 20-25 items are needed for tests until the first failure occurs. For larger acceptable values of  $P_N$  the number of items tested increases significantly, especially if the value of "p" exceeds 0.9.

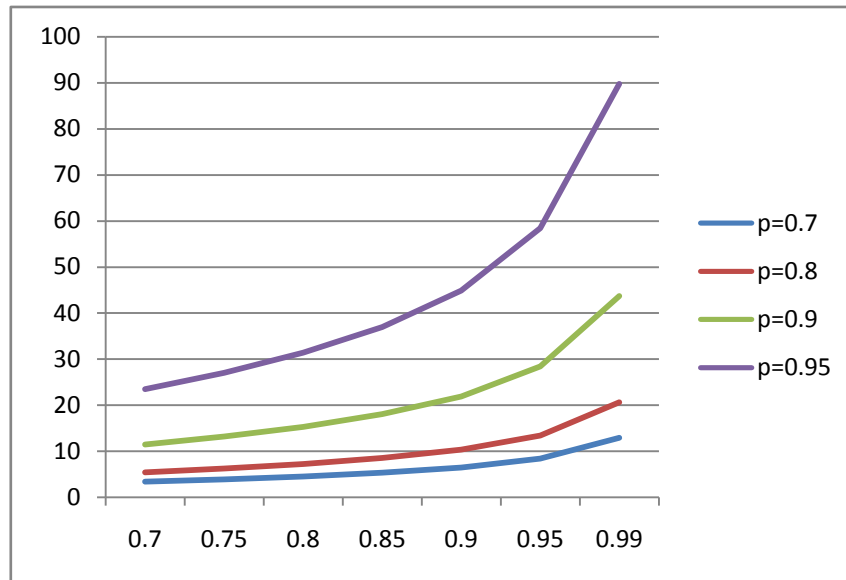


Figure 3 Dependency of N from  $P_N$  for some typical values of "p"

Two results are possible when conducting this type of experiment. The first result concerns the occurrence of a failure before the total amount of N test products is exhausted - when testing the "M<sub>1</sub>" item ( $M_1 < N$ ). In this case, it is possible to terminate the test or continue with the other items of the test series.

If the test is terminated when the first failure occurs precisely when testing the "M<sub>1</sub>" item, then the veracity of this probability of absence of a defect in the batch being equal to "p" is less than the set value ( $P_N$ ). Considering that the distribution density of this event is subordinate to the geometric probability ( $P_{M_1}$ ) can be calculated as :

$$(12) \quad P_{M_1} = (1 - p)p^{(M_1-1)}$$

If  $P_{M_1}$  satisfies the manufacturer (Stoichev K., D. Bratanov, S. Burdzhiev, D. Dimitrov, V. Panevski, G. Georgiev, G. Damjanov.2014, Stoichev K..2014), the tests may be terminated, otherwise they may continue. If, then, in the test of the "M<sub>2</sub>" item ( $M_2 < N$ ) a second failure

occurs, the probability of two failures ( $P_{M_2}$ ) when testing the "M<sub>2</sub>" item is described with the distribution of Pascal and can be calculated by the formula :

$$(13) \quad P_{M_2} = C_{(M_2-1)}^1 (1-p)^2 p^{(M_2-2)}$$

If the probability value  $P_{M_2}$  satisfies the manufacturer, tests may be terminated, otherwise they may continue. Thus, in the event of faults occurring before the tests of the whole series of "N" number of products, the tests may continue until a satisfactory probability value "j-th" failure, calculated after the test of the "M<sub>j</sub>" items ( $M_j \leq N$ ).

In accordance with Pascal law, the probability "P<sub>j</sub>" is determined by the formula [2,4]:

$$(14) \quad P_{M_j} = C_{(M_j-1)}^{(j-1)} (1-p)^j p^{(M_j-j)}$$

If after testing the whole series of "N" products there is no failure, then obviously with a probability greater than  $P_N$  it can be assumed that the value of "p" is not less than the predicted value.

## CONCLUSION

This paper examines the problems related to the supply of additional components necessary for the production of articles in limited quantities. A method is proposed to optimize the quantity of delivery components, applying the minimum financial risk criterion. Also proposed is a method for determining the amount of items required to perform tests to verify their reliability.

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