International Journal of Economics, Commerce and Management
United Kingdom Vol. IV, Issue 6, June 2016
http://ijecm.co.uk/ ISSN 2348 0386

FOUNDATIONS OF EXPECTED UTILITY THEORY AND ITS ROLE IN THE PURCHASE OF INSURANCE

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Abstract

Decision making is a function of an individual's preference. In the midst of uncertainty a decision maker will aim to maximize his utility. This paper reviews the bedrock upon which the expected utility theory (EUT) came to bear, notably, the famous St. Petersburg Paradox posed by the Swiss mathematician Daniel Bernoulli. The foundational axioms of decision theory on which the EUT relies, is presented and the EUT is derived. Furthermore, a typical case scenario between a risk-averse consumer and the market for risk (insurance) is considered. It is shown that EUT plays a major role in determining whether or not the consumer will purchase insurance. The implication of such findings is reflected in the pricing of the insurance product. A Monte Carlo simulation is carried out to show how the degree of risk aversion affects insurance pricing. Simple illustrations are frequently used in this paper to help explain in simple terms the implications associated with making different choices.

Keywords: Expected Utility Theory, St. Petersburg Paradox, Decision Theory, Insurance Pricing, Monte Carlo Simulation

INTRODUCTION

Conditions of uncertainty surround us all the time. Despite these conditions, man must still make decisions bordering on his economy, environment, health, relationship with others and so on. Choosing between two, possibly equally likely options, brings to fore the importance of the expected utility theory.

Why Expected Utility Theory?

Expected utility theory is basically about individuals making rational choices when they are not sure of which outcome will result from their actions. That is, the behavior of people under uncertainty based on their perception.

Definition: Utility refers to the satisfaction one gets from something or the measure of how useful something is.

How Much Is A Risky Decision Worth?

We can refer to a risky decision as a gamble i.e. a finite set of outcomes with their respective probabilities.

Expected value (EV) provides us with an answer to the question posed above (1.2).

Illustration: We illustrate this concept with a simple example and show that one should be willing to pay the expected value of a gamble in order to participate in the gamble (or take on the risky decision).

EV = Sum of all possible outcomes of a decision(or action) multiplied by their respective probabilities

i.e. $E(x) = \sum_{i=1}^{n} xP(x)$

Example 1

Say you are faced with the opportunity to buy a new technology for $200 million and you know that there is a 20% chance it will earn you $500mil, a 40% chance will earn it will earn you $200miland a 40% chance it will earn you $0 (it becomes worth less)

Would you take on the investment or not? (mil=million)

Solution

We first compute the EV

EV = 500mil x 0.2 + 200mil x 0.4 + 0 x 0.4 = 180mil

We note that $180mil is less than 200mil (initial investment)

Decision

It would not make sense for you to take the risk of buying the technology.

THEORETICAL REVIEW

Expected Utility Model

The expected utility model dates back to the 18th century with the work of Daniel Bernoulli on the utility function and the measurement of risk. In his work, he presents the concept of declining marginal utility and its effect on decision making.

Thiago G. M. (2014) in simple terms draws out important aspects of Bernoulli's work: "Prior to this work, it was assumed that decisions were made on an expected value or linear utility basis. Bernoulli then developed the concept of declining marginal utility, which lead to the logarithmic utility. The general idea of declining marginal utility also referred to as "risk aversion" or "concavity" is crucial in modern decision theory. He criticized the notion of linear utility. He (Bernoulli) goes on to redefine the concept of value to a more general one. "The determination of the value of an item must not be based on its price, but rather on the utility it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate. Thus there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount."

Bernoulli further postulates that "it is highly probable that any increase in wealth, no matter how insignificant, will always result in an increase in utility which is inversely proportionate to the quantity of goods already possessed." That is, he not only presented the notion of declining marginal utility but also proposes a specific functional form namely the logarithmic utility function:

$dw=w^{-1}dw \rightarrow u(w)=\ln w$

The conclusion then, is that a decision must be made based on expected utility rather than on expected value."

Expected utility v/s Expected Value

According to Lengwiler Y. (2009), expected utility theory consists of two components. The first component is that people use or should use the expected value of utility of different possible outcomes of their choices as a guide for making decisions. The second component is the idea or insight that more of the same creates additional utility only at a decreasing rate. With "expected value" we mean the weighted sum, where the weights are the probabilities of the different possible outcomes.

Therefore in simple terms, we can say that expected value is an absolute term, meaning that the correct choice is the same for every one whereas expected utility is a relative term, i.e. the correct choice for one person is not necessarily the correct choice for another person.

Illustration: Marciszewski W. (2003) captures an intuitive way of thinking about expected utility and expected value as shown below.

Example 2

Given two options

Option 1: probability of $Y is P%

Option 2 : probability of $Z is q%

where Y, Z, p and q are clearly specified.

Expected value will state:-

a) you should choose option 1
b) you should choose option 2
c) you should be indifferent between options 1 and 2

That is, any one option amongst the 3 will be right for all persons.

On the other hand,expected utility will state:-

Depending on the utility you assign to 1 and 2

a) you might choose option 1
b) you might choose option 2
c) you might be indifferent between options 1 and 2

This implies that the choice made is dependent on the satisfaction it provides the specific person.

Hence, we see a conflict arising between an individual's personal choice and the expected value theory. This problem was first observed by a Swiss mathematician Daniel Bernoulli in the 18th century precisely in1738. He was puzzled by the fact that for a game which offered an infinite expected payoff, only few participants were willing to pay more than a moderate amount to participate in the game.

This problem which is known as the *St. Petersburg Paradox*laid the foundation upon which the expected utility theory was built.

The St. Petersburg Paradox

The St. Petersburg game is based on flipping a fair coin. A fair coin implies that there is a 50:50 chance of it landing on its head or tail.

The game is described below.

- Flip a fair coin continuously
- when the first head appears
- the player stops playing.

The payout (*x*)

- If the first toss is a head, the player gets $2
- If the first head comes up on the second flip, the player is paid $4 and so on.

Hence, if first head comes up on the nth flip, the player is paid $$2^n$

Table1: Below Summarizes the Outcomes and Payouts of the Game

Outcome	Payout (*x*)	Probability (p)
H	2^1	½
TH	2^2	$(1/2)^2$
TTH	2^3	$(1/2)^3$
…	…	…

The expected value of the St Petersburg game would be

E(*x*) = $\sum_{i=1}^{n} xP(x)$

= ½ x 2 + $(1/2)^2$ x 2^2 + $(1/2)^3$ x 2^3+………

=1 +1 +1 +1 +1+……=$\sum_{i=1}^{\infty} 2^m \, (1/2)^m$ = ∞

The expected value of the game is infinite.

The Paradox

Based on the assumption that an individual will maximize expected monetary value, it implies that a person should be more than willing to pay any finite amount of money to play the game because the amount will be less than the expected value of the winnings.

Here is where the paradox occurs: individuals are not willing to pay any finite or unlimited amount of money to play this game.

Bernoulli found out that most people would not pay more than a reasonable amount of $20 to participate.

Why? What price can be termed the fair price of the game?

Bernoulli's Solution

Bernoulli proposes a simple solution to the problem. It is rare that rather simple concepts underlie such a puzzling problem, Gracia J. (2013). According to Bernoulli, individuals maximize their expected utility from a gamble and not the expected value of the gamble.

This can simply be explained by noting that since an individual's utility is the perceived value he attaches to something, it would imply that the individual's utility will be affected by his current status. Hence, a very poor person would attach more value to a $100 bill than a rich person.

Bernoulli asserted that an individual's utility from money was a logarithmic function of the amount of money. He used the equation:

U(*w*) = log*ww* = dollar payout

The idea behind his use of the log function lay in the fact that the value of a log function increases at a decreasing rate, as the value of its argument increases. This gives a concave shape.

Table 2: Presents the Expected Utilities for each of the Payoffs

	Probability	Pay off ($*w*)	Utility payout: *log(w)*	Expected utility of payment: P(*log(w)*)
1	1/2	2	0.301	0.1505
2	1/4	4	0.602	0.1505
3	1/8	8	0.903	0.1129
4	1/16	16	1.204	0.0753
5	1/32	32	1.505	0.0470
6	1/64	64	1.806	0.0282
7	1/128	128	2.107	0.0165
8	1/256	256	2.408	0.0094
9	1/512	512	2.709	0.0053
10	1/1024	1024	3.010	0.0029
11	1/2048	2048	3.311	0.0016
12	1/4096	4096	3.612	0.0009
13	1/8192	8192	3.913	0.0005
14	1/16384	16384	4.214	0.0003
15	1/32768	32768	4.515	0.0001
16	1/65536	65536	4.816	0.0001

From the table we note that as utility of payout increases, expected utility of payout falls

Summing the expected utilities we get:$\sum_{i=1}^{n} P(\log(w))$= 0.6019.

From the 15th row, the expected utility is approximately zero so it doesn't really affect the sum. The sum of the expected utilities corresponds to $4(antilog $10^{0.60206}$). This means that an individual whose utility is represented by the log function will be willing to pay up to $4 to take part in the game.

This is the solution that was proposed by Bernoulli. While some were satisfied by this explanation, others were not. Bernoulli's formulation was not perfect - the paradox becomes a paradox once again if we raise the payoffs to, say, $\$10^{2n}$. Finally, in 1944, mathematicians John Von Neumann and Oskar Morganstern derived a theory of expected utility that resolved many of the peculiarities related to people's behavior under uncertainty, EconPort (2006).Martin R.(2014) puts it this way:

'Some have found this response to the paradox unsatisfactory, because Bernoulli's association of utility with the logarithm of monetary amount seems way off. On his scale, the utility gained by doubling any amount of money is the same; thus the difference in utility between $2 and $4 is the same as the difference between $512 and $1024. However there are other ways of discounting utility as the total goes up which may seem more intuitively plausible (see for example Hardin 1982; Gustason 1994; Jeffrey 1983). The main point here is that if the sum of the utilities in the right-hand column approaches a limit, then the St. Petersburg problem is solved. The rational amount to pay is anything less than this limit. In his classical treatment of the problem, Menger (1967 [1934]) argues that the assumption that there is an upper limit to utility of something is the only way that the paradox can be resolved.'

AXIOMS UNDERLINING THE UTILITY THEORY

There are four main axioms.

Completeness

This means that an individual must be able to state a preference between certain outcomes.

In other words, an individual has well defined preferences and given any two alternatives A and B*either* $A \geq B$ *or* $B \geq A$

$A \geq B$ is read as A is strictly preferred or indifferent to B (where ≥ is the utility preference representation).

Transitivity

This implies that an individual decides consistently according to the completeness axiom. Hence, for any 3 gambles A, B and C, *if* $A \geq B$ *and* $B \geq C$ *then* $A \geq C$

Independence

This refers to the independence of irrelevant alternatives.

That is, given 2 gambles (*X, A, P*) and (*X, B, P*), $A \geq B$ *if* $(X, A, P) \geq (X, B, P)$

i.e. $p(A) + (1-p)X \geq p(B) + (1-p)X$

We see that the preference order of the 2 gambles mixed with the third one [*(X, A, P) , (X, B, P)*] is the same as the preference order of the 2 gambles independent of the third one [*A, B*]

Here *X* is irrelevant. Its inclusion does not change the preference of the investor about A and B.

Note: (*X, A, P*) means a gamble that pays *X* with probability p and A with probability (1-p)

Continuity

This axiom states that given 3 gambles A, B and C, if A ≥ B and B≥ C holds then there should be a possible combination of A and C such that there exist some probability that the individual is indifferent between the mix of A and C on one hand and the gamble on the other . This implies that a probability exist such that the decision maker is indifferent between the best and the worst occurrence.

DERIVING THE EXPECTED UTILITY

To prove the expected utility theory we make use of the theorem which states that:

A preference relation on P with finite alternative space that satisfies the continuity and independence axioms admits an expected utility representation.

Thus given a finite number of outcomes, the best and the worst options exist. We need to proof that

i) The independence axiom is satisfied.

ii) The continuity axiom is satisfied and most importantly,

iii) An expected utility function is linear in the probabilities. Meaning that for L, L* ϵ SL and α ϵ(0, 1) we will have U[αL+(1- α)L*]= αU(L)+(1- α)U(L*). It should be noted that for a utility function to have an expected utility form, it must be linear in the probabilities.

This proves the expected utility representation (E[u(w)]).

The Proof

Faced with a finite number of outcomes, it implies that a 'best' and a 'worst' outcome exist. Hence, we can assume that the best and the worst lottery exist on the space of lotteries. We denote best lotteries by L**u** and worst lottery by L**d**. This means that L**u ≥** L**d.**

i) To show that the independence axiom is satisfied.

Now, for any two lotteries L*and L

a) If L > L*and

b) given probability p where p ϵ (0, 1) then,

by the independence axiom it follows that for L,L* and L"

L > L*if and only if (iff)p(L) + (1-p) L"> p(L*) + (1-p) L"

(>means 'strictly preferred to')

We note that:

L = p(L) + (1-p)L > p(L) + (1-p)L*> p(L*) + (1-p)L*= L*

This implies that L >p(L) + (1-p)L* > L*

So if we have p and p*ϵ (0, 1) then,

p(L**u**) + (1-p) L**d**> p*(L**u**) + (1-p*) L**d** if fp > p*.

This satisfies the independence condition.

ii) To show that the continuity axiom is satisfied.

From the continuity axiom given L**u**, L, L**d**, there exists a unique probability pL such that

pL (L**u**) + (1- pL)L**d**~ L …………….(1) (~ means 'indifferent to')

Let us consider the following 2 sets:

p (L**u**) + (1- p) L**d**≥ L withp ε[0,1] and L ≥ p (L**u**) + (1- p) L**d** , p ε [0,1] .

Based on the fact that ≥ is continuous and L**u**, L**d** are the best and the worst lotteries respectively, it implies that both sets are closed and if we choose any p, it will belong to at least one set. We note the following: the unit interval is connected and both sets have at least an element. Therefore we can find at least a p=pL such that (1) holds.

iii) To show that an expected utility function is linear in the probabilities

Let U:SL→R which assigns U(L)= pLfor all L ϵ SL, pLϵ R.

(SL is the set of all possible lottery outcomes and R is the set of real numbers).This implies that, U represents the preference relation ≥ .This follows from the fact that L > L*iffpL>pL*.

Now we show that forL, L*ϵSL and α ϵ(0, 1) we will have

U[αL+(1- α)L*]= αU(L)+(1- α)U(L*)

We proceed:

From (1)L~ pL (L**u**) + (1- pL) L**d=** U(L) L**u** +(1- U(L)) L**d** ………(2)

Similarly,

L* ~ pL* (L**u**) + (1- pL*) L**d=** U(L*) L**u** +(1- U(L*)) L**d** ………(3)

Multiplying (2) by α and (3) by (1- α) and summing we get

α L~ α [U(L) L**u** +(1- U(L)) L**d**]

(1- α) L* ~ (1- α) [U(L*) L**u** +(1- U(L*)) L**d**]

α L+ (1- α) L* ~α [U(L) L**u** +(1- U(L)) L**d**] + (1- α) [U(L*) L**u** +(1- U(L*)) L**d**]

→ α L+ (1- α) L* ~[α U(L) +(1- α) U(L*)] L**u** +[α (1-U(L)) +(1- α) (1-U(L*))]L**d**

=[α U(L) +(1- α) U(L*)] L**u** +[1- αU(L) +(α-1)U(L*)]L**d**

=[α pL +(1- α) pL*] L**u** +[1- α pL +(α-1) pL*]L**d** (since U(L) = pL)

= α pL L**u**+(1- α) pL*L**u** + L**d** - α pLL**d** +(α-1) pL*L**d**

= α pL L**u** + pL*L**u** - α pL*L**u** + L**d** - α pLL**d** +α pL*L**d** - pL*L**d**

{We introduce(αL**d** -αL**d**)into the equation}

= αL**d** - αL**d** +α pL L**u** + pL*L**u** - α pL'L**u** + L**d** - α pLL**d** +α pL*L**d** - pL*L**d**

= α pL L**u** + αL**d** - α pLL**d** +pL*L**u** + L**d** - pL*L**d** -α pL*L**u**- αL**d**+α pL*L**d**

$= \alpha [pL\ L\mathbf{u} + (1- pL)L\mathbf{d}] + pL^*L\mathbf{u} - \alpha\ pL^*L\mathbf{u} + \alpha pL^*L\mathbf{d} - pL^*L\mathbf{d} + L\mathbf{d} - \alpha L\mathbf{d}$

$= \alpha [pL\ L\mathbf{u} + (1- pL)L\mathbf{d}] + [(1 - \alpha)\ pL^*L\mathbf{u} + \{(1-\alpha) - (1-\alpha)pL^*\}L\mathbf{d}]$

$= \alpha [pL\ L\mathbf{u} + (1- pL)L\mathbf{d}] + (1 - \alpha) [pL^*L\mathbf{u} + (1- pL^*)\ L\mathbf{d}]$

Now we have

$\alpha L + (1- \alpha) L^* \sim \alpha [pL\ L\mathbf{u} + (1- pL)L\mathbf{d}] + (1 - \alpha) [pL^*L\mathbf{u} + (1- pL^*)\ L\mathbf{d}]$

Since they are indifferent, it implies that their utility should be equal

$U[\alpha L + (1- \alpha) L^*] = U[\alpha [pL\ L\mathbf{u} + (1- pL)L\mathbf{d}] + (1 - \alpha) [pL^*L\mathbf{u} + (1- pL^*)\ L\mathbf{d}]]$

$= \alpha\ U [pL\ L\mathbf{u} + (1- pL)L\mathbf{d}] + (1 - \alpha)\ U[pL^*L\mathbf{u} + (1- pL^*)\ L\mathbf{d}]$

$= \alpha\ U (L) + (1 - \alpha)\ U(L^*)$ (from equations 2 and 3)

This demonstrates the fact that a utility function which satisfies the axioms of independence and continuity is also linear in nature. This means that it exhibits the expected utility property,

$U[\alpha L + (1- \alpha) L^*] = \alpha\ U (L) + (1 - \alpha)\ U(L^*)$

So far we have seen that:

1. Individuals exhibit preferences
2. Utility measures satisfaction. It is used to compare two or more options faced by an individual.
3. Thus, given that individuals always aim to maximize their satisfaction, it means that they aim to maximize their utility (in situations that are certain).
4. In uncertain situations, expected utility is applied to help the individuals make choices as the face uncertain outcomes. We can therefore say that individuals aim to maximize expected utility under uncertainty.
5. Understanding an individual's attitude towards risk is important as it will play a major role in the decision making process of the individual. This decision process arises when we consider a fair gamble.

- A risk-averse individual will refuse to participate in a fair gamble.
- A risk neutral person will be indifferent to participation in a fair game.
- A risk loving person will be prepared to pay for the right to participate in a fair game.

It is generally assumed that individuals are risk averse (which implies that they possess a decreasing marginal utility of wealth) and they want to maximize expected utility.

Given that a risk-averse investor will always want to hedge against risks, it serves as a basis for the transfer of risks. The more an individual is risk averse the more he will be willing to pay to hedge his risk.

Insurance deals with the transfer of risk. Hence the demand for insurance hinges on how much an individual is averse to risk. Thus an individual's degree of risk aversion is a vital component in the pricing of insurance and in the decision to purchasing insurance.

EXPECTED UTILITY AND INSURANCE

An area where expected utility theory finds applications is in insurance sales. Insurance companies take on calculated risks with the aim of long-term financial gain. They must take into account the chance of going broke in the short run, Briggs R (2014). By making use of a simple illustration, we show the role expected utility plays in determining

1. whether one should purchase insurance or not
2. the price of the insurance

To Purchase Insurance or not to Purchase

The example used by Etti B. et al (2012), serves as a guide:

Consider a risk averse consumer having an initial wealth of $200 and a utility function of $U(w) = \sqrt{w}$

(NB:$U(w) = \sqrt{w}$is concave in shape which represents the attitude of risk aversion.)

Suppose there is a 50% probability that he would get into an accident and lose $100 on any given day and a 50% chance that he will not lose nothing i.e no accident occurs.

a. What is the expected amount of money he would lose?

$E(L) = 0.5 \times 100 + 0.5 \times 0 = \$50$ (L=loss)

b. What is his expected wealth?

$E(w) = 0.5(200-0) + 0.5(200-100) = \$150$

c. Since the consumer is risk averse he will want to hedge the risk of having a loss of $50.How much will he be willing to pay to afford him peace of mind in the event of an accident taking place?

We apply the expected utility theory to help us answer this question.

i. What is his expected utility if he decides not to insure the risk?

$E(U) = \sum U(w)\, P(w) = 0.5U(200) + 0.5U(100) = 0.5\sqrt{200} + 0.5\sqrt{100} = 12.071$

ii. If he decides to insure the risk at what price will that be?

We note that since his expected loss is $50, he will want to hedge the risk at the same price

Thus the actuarially fair premium (AFP) would be equal to his expected loss = $50

iii. Does hedging at the AFP increase or decrease his expected utility

If he pays $50 to insure the risk of loss and he doesn't suffer any loss, his final wealth will be $200 - $50 = $150.

However if he suffers a loss, he will be indemnified by the insurance company and he will be brought back to his status. Therefore, his final wealth will be:

w**o** – (AFP) – L + indemnity = 200– 50 = 150

-100 (L) =50

+ 100 (indemnity) = $150

Thus, his expected utility is

0.5U(150) + 0.5U(150) = 2 x 0.5 x √150 = √150 = 12.247

We know that the consumer aims to maximize his expected utility. Therefore hedging provides a greater expected utility than not hedging. His expected wealth remains unchanged i.e. utility rises with the purchase of insurance.

Decision: He will fully hedge his risk at the actuarially fair premium i.e. he will obtain full insurance at AFP. Full insurance implies that the insurance company will cover all his losses.

d. What is his certainty equivalent (CE) wealth?

This is the certain amount of money that will provide him with the same expected utility as his initial (uncertain) position or situation.

U(wCE) = 12.071

√wCE =12.071

wCE=12.071^2 =$145.71

This means that the consumer is indifferent between receiving $145.71 and facing his uncertain situation.

We note that in reality, at the AFP, the insurance company will not be able to cover its cost (y) because

Profit = insurance premium – expected payout- operating costs= 50-50-y=-y

This means that they will be working at a loss. To avoid losses, a charge that is higher than the AFP is applied.

e. If the premium is greater than the AFP, what will be the maximum amount the consumer will be willing to pay to obtain full insurance? This is given by

$w_o - w_{CE}$ = 200 – 145.71= $54.29

alternatively,

If X is the maximum the consumer is willing to pay, then

U(200-X)=E(U) if he does not insure

√ (200-X)= 12.071 therefore X= $54.29

f. What is his risk premium?

The risk premium is the price in the excess of the AFP that their consumer is willing to pay to hedge the risk.

Rpremium = 54.29 - 50 = $4.29

Analysis of the above Example

- ✓ The consumer begins with an initial wealth of $200.
- ✓ Paying $54.29 to the insurance company to cover his losses (in the event of an accident) will leave him with a final wealth(his certainty equivalent wealth) which is $145.71. This is what he gets regardless of the situation he faces.
- ✓ Obtaining $145.71 for certain or having to face his initial uncertain position will give him the expected utility of 12.071. Therefore paying for full insurance at the rate of $54.29 will not really be worth it.
- ✓ The consumer realizes that at the rate of the AFP (i.e. $50) he will be able to maintain same level of wealth in all states (i.e. S_o= uncertain state and S_1 = insured state). On the other hand, the insurer also realizes that at the AFP rate, he will make losses. This knowledge plays a major role in the pricing of the insurance product.

Pricing the Insurance Product

The insurer has to price his product in such a way that he makes the consumer willing to buy and at the same time he makes allowance for some profit. To increase the demand for his product, he will need to reduce the price of the insurance. More specifically, his charges should range between the AFP and maximum premium the consumer is willing to pay-$53.29 for example.

This will increase the expected utility of the consumer making the product more attractive to him although this option does not ensure full insurance coverage. Conversely, increasing the premiums, dips the expected utility of the consumer which makes it more difficult for him to purchase the insurance product.

The Methodology: Monte Carlo Simulation

We take a look at how the degree of the risk-averse nature of a consumer affects the pricing of the insurance product. A Monte Carlo simulation of a pool of 1000 drivers is run with varying degrees of risk-aversion. John M. (2014) gives an example that is expatiated upon here.

Assumption: the risk of an accident of any one driver is independent of any other driver's risk. That is, the risks are independent.

The following degrees are used: 1%, 2.5% and 5%.

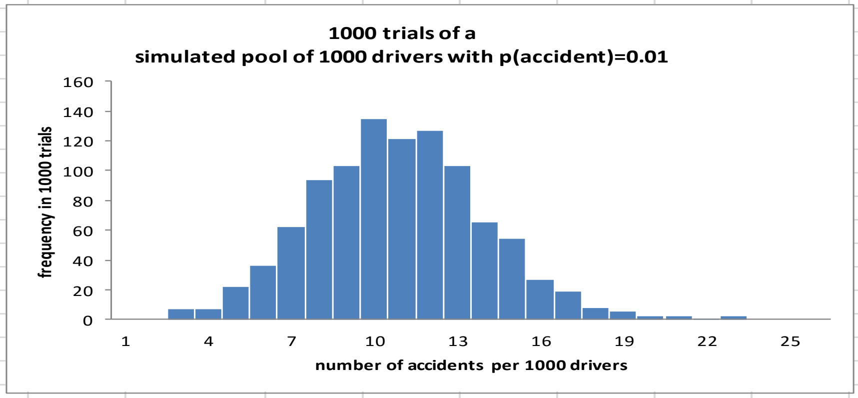
With the probability of accident being equal to 1%, a uniform distribution between 0 and 1 is generated using the RAND() function. In order to represent a random driver with a 99% likelihood of having a zero (no accident), and a 1% probability of getting a one (accident), the formula INT (RAND()+0.01) is used and copied 1000 times. The data table function in EXCEL is used to replicate the simulation model 1000 times. The sum represents the total number of claims in each run. The number of claims changes each time the model is replicated, and all the simulated values vary randomly around an expected value of 10. The minimum number of accidents (min), maximum number of accidents (max), mean and standard deviation (stdev) values are also computed for each run. Fig 1a shows the result of a cross section of one simulated run. Other probabilities are computed in similar fashion.

Figure 1a: A Cross Section of the Monte Carlo Experiment

		0			**sum**	**min**	**max**	**mean**	**stdev**
		0			46	0	1	0.045908	0.20939
		0			54	0	1	0.053892	0.225918
		0			53	0	1	0.052894	0.223934
		0			35	0	1	0.03493	0.183695
prob(accident)		0			50	0	1	0.0499	0.217847
0.01		0			49	0	1	0.048902	0.215771
0.025		0			46	0	1	0.045908	0.20939
0.05		0			53	0	1	0.052894	0.223934
0.075		0			34	0	1	0.033932	0.181145
		0			54	0	1	0.053892	0.225918
		0			56	0	1	0.055888	0.22982
		0			51	0	1	0.050898	0.2199
		0			45	0	1	0.04491	0.20721
		0			53	0	1	0.052894	0.223934
		0			45	0	1	0.04491	0.20721
		0			42	0	1	0.041916	0.200498
		0			61	0	1	0.060878	0.239226
		0			51	0	1	0.050898	0.2199
		0			47	0	1	0.046906	0.211544
		0			60	0	1	0.05988	0.237383
		0			45	0	1	0.04491	0.20721
		1			45	0	1	0.04491	0.20721
		0			58	0	1	0.057884	0.233641
		0			50	0	1	0.0499	0.217847
		0			36	0	1	0.035928	0.186204
		0			55	0	1	0.05489	0.22788
		0			36	0	1	0.035928	0.186204
		0			51	0	1	0.050898	0.2199
		0			46	0	1	0.045908	0.20939
		0			48	0	1	0.047904	0.21367
		0			53	0	1	0.052894	0.223934
		0			57	0	1	0.056886	0.231741
		0			42	0	1	0.041916	0.200498
		0			45	0	1	0.04491	0.20721
		0			57	0	1	0.056886	0.231741
		0			54	0	1	0.053892	0.225918
		0			55	0	1	0.05489	0.22788
		0			57	0	1	0.056886	0.231741
		0			46	0	1	0.045908	0.20939
		0			56	0	1	0.055888	0.22982
		0			59	0	1	0.058882	0.235522

Figure 1b: A Simulated Result Representing the Pool of 1000 Drivers with the Least Degree of Risk-Aversion

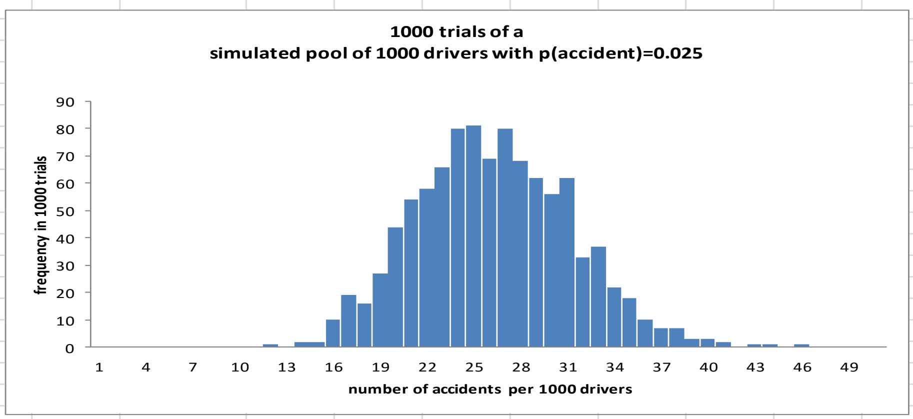
# of accidents	frequency of claims
0	0
1	0
2	7
3	7
4	22
5	36
6	62
7	94
8	103
9	135
10	121
11	127
12	103
13	65
14	54
15	27
16	19
17	8
18	5
19	2
20	2
21	1
22	2
23	0
24	0
25	0



mean of trials	9.874251
stdev of trials	3.110062

Figure 1c: A Simulated Result Representing the Pool of 1000 Drivers with the Degree of Risk-Aversion=2.5%.

# of accidents	frequency of claims
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	1
12	0
13	2
14	2
15	10
16	19
17	16
18	27
19	44
20	54
21	58
22	66
23	80
24	81
25	69
26	80
27	68
28	62
29	56
30	62
31	33

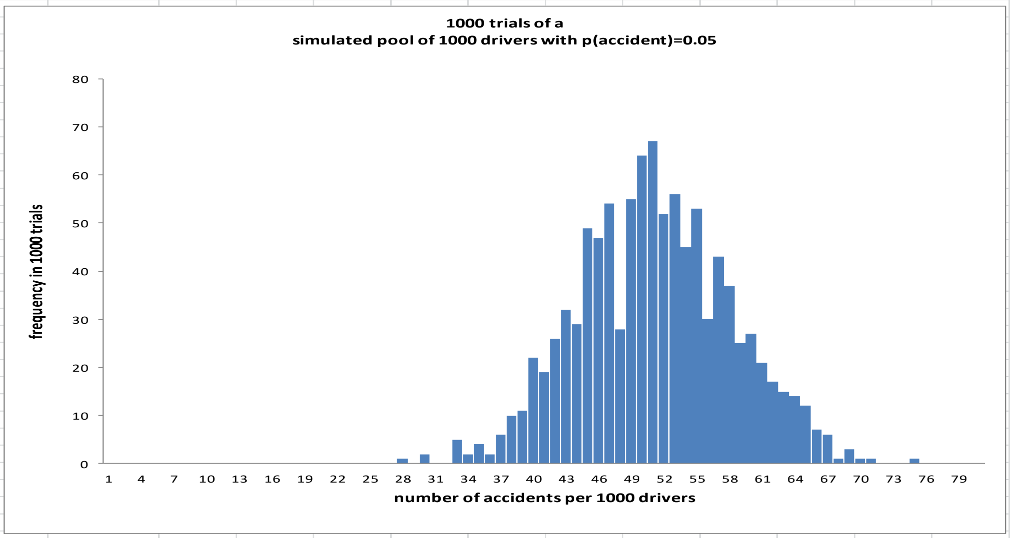


mean of trials	25.30439
stdev of trials	4.998717

Figure 1d: A Simulated Result Representing the Pool of 1000 Drivers with the Highest Degree of Risk-Aversion

# of accidents	frequency of claims
0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	0
19	0
20	0
21	0
22	0
23	0
24	0
25	0
26	0
27	1
28	0
29	2
30	0
31	0
32	5
33	2
34	4
35	2
36	6
37	10
38	11
39	22
40	19
41	26
42	32
43	29
44	49
45	47
46	54
47	28
48	55
49	64
50	67
51	52
52	56
53	45
54	53
55	30
56	43
57	37
58	25
59	27
60	21
61	17
62	15
63	14
64	12
65	7

mean of trials	50.12575
stdev of trials	7.079127



Analysis of the Monte Carlo Experiment

For all cases (of risk-aversion), it is observed that the number of accidents in the whole pool of drivers follows a normal distribution.

In fig1b the simulated result predicts that 1,000 drivers will have mean of approximately 10 accidents per year with a standard deviation of about 3 accidents (*+/-30% of the mean)*. This means that about 68% of the time we would expect between 7 and 13 accidents in this pool of drivers, and between 4 and 16 accidents 95% of the time. The probability that this pool will have more than 16 accidents in a year is only about 2.5%. Simulated result in fig 1cpredicts that 1,000 drivers will have mean of approximately 25 accidents per year with a standard deviation of about 5 accidents (+*/-50% of the mean)*. This means that about 68% of the time we would expect between 20 and 30 accidents in this pool of drivers, and between 15 and 35 accidents 95% of the time. The probability that this pool will have more than 35 accidents in a year is only about 2.5%. In fig1d. the simulated result predicts that 1,000 drivers will have mean of approximately 50 accidents per year with a standard deviation of about 7 accidents (+*/-70% of the mean)*. This means that about 68% of the time we would expect between 43 and 57 accidents in this pool of drivers, and between 46 and 64 accidents 95% of the time. The probability that this pool will have more than 64 accidents in a year is only about 2.5%.

How Does the Varying Degree of Risk Aversion Affect the Pricing of the Product?

Let's say that the average accident loss is $50000 per annum and the aim is to insure each pool of drivers such that we obtain a 97.5% certainty that the premiums paid will cover all losses while taking into account profit for the company. Then for the:

i. Least risk-averse drivers having a probability of accident equals 0.01, the annual premium has to be set to cover 16 accidents. This will be 16*50000= $800000. Thus each driver belonging to this specific pool will have to pay $80 ($800000/1000).
ii. Most risk-averse drivers having a probability of accident equals 0.05, the annual premium has to be set to cover 64 accidents. This will be 64*50000= $3200000. Thus each driver belonging to this specific pool will have to pay $3200 ($3200000/1000).

The amount computed in both cases covers the mean predicted number of accidents plus two standard deviations. Similar computation can be carried out for drivers belonging to the pool represented in fig1c.We note that the annual premium per driver in case ii is larger because of the higher degree of risk aversion.

CONCLUSION

Expected utility theory is the core model that drives decision making under uncertainty. It takes into account the perception of an individual which is an important factor in deciding whether to purchase insurance or not. An individual who is neutral to risk will not see the need to purchase insurance. For an individual who is averse to risk, hedging his risk is vital. The expected utility theory plays a major role in guiding the insurer on how to price the insurance product.

Pricing the insurance product at the actuarially fair premium rate will cause the insurer to make losses while the consumer will maintain same level of wealth at all states prompting him (consumer) to opt for full insurance coverage. This is not a balanced equation. Therefore, to make some profit the insurer will need to increase the premium paid by the consumer, but this increase must not be at the same level of the maximum premium the consumer is willing to pay or surpass it. Within this range we strike a sort of balance-the consumer cannot obtain full insurance and the insurer will not be incurring losses. Both parties will gain.

From the preceding sections the results indicate that beyond this range, the expected utility of the consumer will fall and he will find it difficult purchasing the product. This leads to a fall in the demand for the product which adversely affects the insurance company.

As a further study, the risk pooling mechanism in insurance can further be investigated. Monte Carlo simulation can be applied to the example meant to illustrate the major role the law of large numbers plays in defraying the risk across independent events.

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