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DOES BAYESIAN COURNOT EQUILIBRIUM EXIST IN THE PRESENCE OF INCOMPLETE INFORMATION SCENARIO

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Abstract

This systematic study verifies the existence of Bayesian Cournot equilibrium in the presence of incomplete information case. Our results show that when firms have asymmetric information about the market demand and their cost Bayesian Cournot equilibrium maybe unique or non-existence. We are able to present simple and robust examples of duopolies with these features. We also find some sufficient conditions for existence and uniqueness of Cournot equilibrium in a certain class of industries. Finally, more general results are obtained when negative prices are possible.

Keywords: Bayesian, Cournot, Equilibrium Existence, Market Demand

INTRODUCTION

Detailed and historical validation provides comprehensive empirical evidence on the Cournot model with the case of firms producing homogenous good when there is complete information about demand and production costs.

According to Szidarovssky and Yakowitta (1977) the condition for existence of a Cournot equilibrium under a complete information oligopoly centred on the best response correspondence of the firms with closed grappled and convex values. They posited further that if an inverse demand is a decreasing concave function and firms cost functions are convex then a firms pay off is a concave function of its own output, and following continuity assumptions the Nash theorem yields equilibrium existence.

Another approach at establishing existence of Cournot equilibrium in the case of complete information is subject to the firm's output decisions i.e. the best response correspondence of a firm is decreasing in the output of the other firms. This approach was credited to Novshek (1985) who established the existence of equilibrium concave on the inverse demand based on the assumption of monotonicity and continuity on costs. His condition on the inverse demand requires that the marginal revenue of a firm be a decreasing function of the aggregate output of the other firms, and this implies that firm's outputs are strategic substitutes.

Novshek's pioneering work was spawned by other authors (see Vives (1990), Kukushkin (1994), Amir (1996), which employed lattice – theoretic approach and tools to study conditions for strategic substitutes and for existence of fixed points. Vives (1990) observed that Novshek's condition is equivalent to the cardinal modularity of the payoff functions of each firm in its own output and on the aggregate output of other firms. Amir (1996) demonstrates that log concavity of the inverse demand ensures by itself that the payoff functions are ordinary sub modular. Since both types of modularity imply strategic substitutes, existence of equilibrium in a duopoly can be derived by using Tarski Fixed Point Theorem.

The issue of existence of a pure strategies Bayesian Cournot equilibrium has been neglected in the literature by making strong assumptions. For instance, Gakor (1985), Vives (1984, 1988) and Raith (1999) assume that market demand is uncertain and linear, and also the possibility that negative prices may arise for large outputs. In order not to break the linearity of demand function. Here we can infer that incomplete information is assumed only in firms with linear costs this according to Sakai (1985).

Having gone through literature, the issue of incomplete information on an oligopoly has been largely bypassed. Therefore, this study departs from other studies by concentrating on the oligopoly with incomplete information scenario. To the best of our knowledge, no existing studies in Nigeria has ever looked at the complete and incomplete information case and however we are aware of the Sakai work of 1986 on the incomplete information case with linear cost, we will not only consider the linear cost but also non-liner cost.

And, on this note, the rest of this paper is structured as follows – section 2 describes the set-up. Following this section is section 3 which contains examples of oligopolies with incomplete information for which there is no Bayesian Cournot Equilibrium. Section 4 presents equilibrium existence and uniqueness results in the presence of negative price in oligopolies. The last section summarizes the general results in oligopolies with always non-negative prices.

COURNOT COMPETITION WITH INCOMPLETE INFORMATION

The following assumptions must hold when considered the case of incomplete information case in oligopolies.

- The assumption that there is an industry where a set of firms, N = {1, 2, … n} compete in the production of homogenous good.

- And that there exist uncertainty about market demand and production costs. This state of nature is defined by a finite set Ω together with a probability measure μ on the finite set which implies common prior belief of the firms about the distribution of the realised state.

The private information of firm i E N is represented by a partition Π^i of Ω into a disjoint sets.

And, for any ω E Ω, $\Pi^i (\omega)$ represents information set of *i* given ω i.e. the elements of Π^i which contains ω.

There is also the assumption that μ has full support on Ω i.e. $\mu (\Pi^i (\omega) > 0$ for every i E N and ω E Ω.

We also assume that if $q^i (\omega)$ represents the quantity of good produced by firm *i* instate ωEΩ, and $Q(\omega) = \sum^n_{i=l} q^i (\omega)$ is the aggregate output in ω, then the profit of the firm *i* in ω is given by

$$U^i (\omega, (q^i (\omega), ...q^n (\omega) = q^i (\omega) p (\omega, Q (\omega) – C^i (\omega, q^i (\omega)$$

where P (ω,.) is the inverse demand function in ω and $C^i (\omega)$ is the cost function of firm *i* in ω.

So, also the following three conditions will be considered during our analysis.

i. For every ω E Ω and i E N, $C^i (\omega)$ is continuous and satisfies $C^i (\omega, o) = 0$ which implies there are no fixed costs-----1.

ii. For every ω E Ω, p (ω) is a non-increasing and for every ω E Ω there exist a level of aggregate output Q (ω) E (o, ∞) such that for every Q < Q (ω) p (ω, Q) is truncated and that p (ω, Q (ω) = 0 ----2 and that if Q (ω) < ∞ when Q (ω) is finite. It is referred to as the horizontal demand interception in ω.

iii. There also exist a level of output z < ∞ such that for every i E N and ω E Ω.

$$q\ p (\omega, q) – C^i (\omega, q) \leq 0\text{----}3$$

That is in every state of nature each firm's monopoly profit is non positive when its output exceeds z. If Q (ω) < ∞ for every ω E Ω and the cost function are non-decreasing, then one can maximise z = max ω E Ω Q (ω).

Also, if the monopoly revenue function qp (ω, q) has a maximum and the cost functions are strictly increasing and convex, then such a z exists.

There is also the need to define a pure strategy for firm as a function $q^i: \Omega \rightarrow \mathcal{R}$ which specifies its output in every state of nature, subject to measurability with respects to i's private information (i.e. q^i is constant on every information set of firm *i*).

The set of strategies of firm will be represented by B (Ω, Π^i) given a strategy profile q = (q^i, ...q^n) E $\Pi^n_{j=1}$*B* (Ω, Π^i) the expected profit of firm *i* is U^i (q) = E (U^i(q^i (.), ... q^n (.).

A strategy profile q * E $\Pi^n_{j=1}$*B* (Ω, Π^i) a pure strategy Bayesian Cournot equilibrium, if no firm finds it profitable to unilaterally deviate to another strategy i.e. for every i E N and q^1 E B (Ω, Π^1) such that U^i (*) ≥ U^i (q */ q^i) where (q */ qi) stands for the profile of strategies which is equally identical to q* in all and that E(U^i (; q* (.)) / Π^i (ω) ≥ E (U^i (; (q* / q^i (.)) / Π^i (ω) for every ω E Ω.

Non-existence of Cournot Equilibrium

In this section, we cited two examples of duopolies with incomplete information for which Cournot equilibrium does not exist. The equilibrium non-existence is driven by the asymmetry in firms information about the demand intercept Q. In the following examples, the demand is linear and Q is known to both firms i.e. it is the same in all states of nature.

Example 1: Consider the following duopoly with incomplete information. Here the set of states of nature Ω consist of just two states namely ω_1 and ω_2. The probability of ω_1 is $^1/_5$ and the probability of ω_2 is $^4/_5$. Firm 1 is informed about the realised state of nature. While firm 2 has no information about it i.e. Π1 = {[ω_2], [ω_2]} and Π^2 = Ω. The inverse demand function is given by P (ω_i, Q) = Max (1-b (ω_1) Q 0 where b (ω_1) = $^1/_5$ and b (ω_2) = 1. Thus both P (ω_1;.) and P (ω_2;.) are linear till they reach zero and this is because they are truncated and set equal to zero.

The inverse demand function P is positive on (0, Q (ω_1) and is zero for Q (ω_1), where Q (ω_1) = 5 and Q (ω_2) = 1. The marginal costs of firm 1 are C^1 (ω_1) = 2 and C^1 (ω_2) = $^1/_{100}$. Firm 2 has a constant marginal cost C^2 = $^1/_{100}$ in both states of nature. The fact that marginal revenue of firm 1 in ω_1 is always below its marginal cost.Then, the need to maximise profits with output zero in this state. In order to determine the equilibrium we need to restrict our attention to those strategies of firm 1 – *q^1 E B* (Ω, Π^1) that prescribe producing zero in ω_1; i.e. q^1 can be identified with a scalar x ≡ q^1 (ω_2) E $\mathbb{R}_+$ so also, since firm 2 lacks information about the realised state, a strategy of firm 2, *q^2 E B* (Ω, Π^2) must specify the same output in both states of nature i.e. q^2 can be identified with a scalar y = q^2 (ω_1) = q^2 (ω_2) E $\mathbb{R}_+$.

Having identifying firms' strategies with scalars x, y the incomplete information scenario has been converted to complete information case.

To understand the source of the existence problem, then the need to assume that the demand functions are not truncated, and hence prices may be negative, then the industry is said to have a Cournot equilibrium.

But if the demand functions are truncated, then Cournot equilibrium may not exist. We then proceed by assuming that the inverse demands

P - (ω_i, Q) = 1 – b (ω_i) Q, and for (x, y) E $\mathbb{R}^2_+$ represents by - 4 U^i – (x, y) the corresponding payoff of firm *i* E (1, 2) since firm 1 only produces in ω_2, we have

$$U^1 - (x, y) = {}^4/_5 (P - (\omega_2, x + y) x - {}^x/_{100}$$

The payoff of firm 2 is

$$U^2 - (x, y) = P^x (y) y - {}^y/_{100}$$

Where, P^x (y) = $^1/_5$ (P – (ω_1, y) + $^4/_5$ P – (ω_2, x + y)
is firm 2's "expected residual inverse demand
the firms' reaction functions are
R^1(y) – max {½ ($^{99}/_{100}$ – y), o} and R^2(x) = max ($^{198}/_{325}$ – $^6/_{13}{}^{x,\,0}$) and
therefore (x*, y*) = ($^{99}/_{400}$, $^{99}/_{200}$) represents the unique Cournot equilibrium, and the expected residual inverse demand function, faced by firm 2.
So also, if the analysis are modified to account for the demand truncated impact states we need to see that the expected residual demand faced by firm 2 is now p^x(y) = $^1/_5$ P(ω, y) + 4/5 (ω_2, x + y);

$$Px(y)=p(y) \begin{cases} Px(y), & \text{if } x + y < 1 \\ & \text{if } 1 \le x + y \text{ and } y \le 5 \\ 0, & \text{if } y > 5 \end{cases}$$

Where, P(y) = $^1/_5$ (1 – $^y/_5$)

Note that px(y) is not a concave function.
Firms' payoffs are given for (x, y) E $\mathbb{R}^2_+$ by U^1 (x, y) = $^4/_5$ (P^1 (ω_2, x + y) x – $^x/_{100}$ - - - 5 and U^2 (x, y) = (P^x (y) y – $^y/_{100}$ - - - 6

It should be noted that U^2(x.) is non-quasi-concave, despite the fact that state dependent payoff functions are quasi-concave with two local maxima.
The local maximum of U^2(μ_1.) given $\max_x$ (p^x(y)y – $^y/_{100}$) = $\max_y$ (U^2(x, y) = U^2(x, $\mathbb{R}_2$(x.)) and this is the firm 2's maximum payoff when the price on state ω_2 is zero given by

$$\max_y (p\hat{}(y)\, y - {}^y/_{100}) = {}^{144}/_{625}$$

Therefore, the smallest solution to the equilibrium is

$$U^2 (x, \mathbb{R}^2 (x) = {}^{144}/_{625}$$

The summary, firm 2's lack information about the demand function as depicted here. As a result, the expected revenue of firm 2 is not quasi-concave in its own output and its reaction correspondence is not convex valued. This causes equilibrium non-existence.

Example 2: Consider an industry identical to that of example 1 except for the demand in w which is given here by

$$P(\omega_1, Q) = \begin{cases} 1, & \text{if } Q \leq \frac{99}{100}; \\ 100(1-Q), & \text{if } \frac{99}{100} < Q \leq 1; \\ 0, & \text{if } Q > 1. \end{cases}$$

The demand intercept is now constant, $Q\{\omega_1\} = Q\{\omega_2\} = 1$, and thus known to both firms.
The expected residual inverse demand faced by firm 2, $P^x(y)$, is given by
Note that for $^1/_{100} < x < 1$ the function P^x (y) is not concave on [0, 1], even though the demand intercept is the same in both states. Figure 3a shows the graphs of the state-dependent residual inverse demands in that case, and the expected residual inverse demand.
Let $(x, y)\, \varepsilon \mathbb{R}^2_+$. As in example 1, since firm 1 produces zero in ω_1 its payoff is

$$\max_y\{\left(\frac{1}{4}+\frac{3}{4}(1-x-y)\right)y-\frac{y}{100}\} = \max_{y\in\left[0,\frac{99}{100}\right]}\{\frac{1}{4}y-\frac{y}{100}\}. \tag{8}$$

Firm 2's payoff is

$$P(\omega, Q) = \max\{\alpha(\omega)(\beta(\omega)-Q), 0\},$$

Note that $U^2(^{22}/_{100},.)$ is not quasi-concave. (This stands in contrast to the case of firm 2 being a monopoly: $U^2(0,.)$ is a quasi-concave function, which is, moreover, concave on [0,1].)
The reaction function of firm 1 is also as in example 1, whereas firm 2's reaction function is now for $x\, \varepsilon \mathbb{R}_+$. Here $x = {}^{33}/_{25} - {}^{6}/_{25}\sqrt{22} \approx 0.194\,3$ is the smallest solution of the equation

In both examples 1 and 2, the demand functions can be made smooth, and such that they do not intersect the horizontal axis in either state, while preserving the form of firm 2 expected profit function. These examples are suggestive of the difficulty in finding natural conditions on the primitives of the model, analogous to those found for the complete information case, that guarantee existence of a Cournot equilibrium when information is incomplete.

Example 3: (A duopoly with a linear demand and complete information on the demand intercept). Suppose that n = 2. Let α, $\beta : \Omega \to \mathbb{R}_{++}$ be strictly positive functions. Assume that $\beta \,\varepsilon\, B(\Omega, \Pi^1) \cap B(\Omega, \Pi^2)$, where Π^1 and Π^2 are information endowments of the duopolists. Suppose that for any ω ε Ω.

And, that the cost functions satisfy (i) and are non-decreasing. Here Q = β. Since β is both Π^1- and Π^2- measurable, both firms know the demand intercept in every state of nature. This is a crucial difference with example 1, where Q was not measurable with respect to the information partition of firm 2, and a Cournot equilibrium does not exist. Here, the measurability Q = β with respect to both partitions leads to a different conclusion.

Let $q^1 = q^2 \equiv \frac{1}{2}\beta \,\varepsilon\, B(\Omega, \Pi^1) \cap (\Omega, \Pi^2)$. Clearly (q^1, q^2) satisfies (12) of condition *C*. But ½ β is the revenue maximizing monopoly output level, since the firms know β and the demand is linear on $[0, \beta]$, and thus no firm will exceed ½β in any best response. Therefore condition *C* holds, and the duopoly has a Cournot equilibrium by theorem 2A.

Example 4: (Non-Uniqueness of Cournot Equilibrium when no Firm Has Superior Information). Consider a duopoly in which 2 consists of three states, ω_1, ω_2, and ω_3; each one is chosen by nature with equal probability. Firms' information partitions are $\Pi^1 = \{\{\omega_1, \omega_2\}, \{\omega_3\}\}$, and $\Pi^2 = \{\{\omega_1, \omega_3\}, \{\omega_2\}\}$; i.e., firm 1 cannot distinguish between ω_1 and ω_2, and firm 2 cannot distinguish between ω_1 and ω_3. In all states of nature firms face the same quadratic inverse demand function

$$P(Q) = \max\{1 - Q^2, 0\}$$

Thus, firms know the inverse demand in every state of nature, but have incomplete information about their costs. Firm 1 has a constant marginal cost of $^1/_{100}$ in states ω_1 and ω_2, while its marginal cost is 2 in ω_3. Firm 2 has a constant marginal cost of $^1/_{100}$ in states ω_1 and ω_3, while its marginal cost is 2 in ω_2.

Since in ω_3 the marginal revenue of firm 1 is always below its marginal cost, firm 1 produces zero in this state in any best response. Similarly, firm 2 produces zero in ω_2 in any best response. It follows that each firm *i*'s strategy q^i can, without loss of generality, be identified with a scalar: q^1 can be viewed as the quantity *x* produced by firm 1 in state ω_1 (and thus also in ω_2) and q^2 as the quantity *y* produced by firm 2 in state ω_1 (and thus also in ω_3).

We claim that both

$$q_* = (x_*, y_*) = \left(\frac{3}{10}\sqrt{2}, \frac{3}{10}\sqrt{2}\right) \approx (0.424\,26, 0.424\,26)$$

$$q_{**} = (x_{**}, y_{**}) = \left(\frac{7}{30}\sqrt{6}, \frac{7}{30}\sqrt{6}\right) \approx (0.571\,55, 0.571\,55)$$

and

are Cournot equilibria.

$$\begin{aligned} U^2(x_*, y) &= \frac{1}{3}u^2(\omega_1, (x_*, y)) + \frac{1}{3}u^2(\omega_2, (x_*, 0)) + \frac{1}{3}u^2(\omega_3, (0, y)) \\ &= \frac{1}{3}y\left(1 - \left(\frac{3}{10}\sqrt{2} + y\right)^2\right) + \frac{1}{3}y\left(1 - y^2\right) - \frac{2}{3}\frac{y}{100}, \end{aligned}$$

Let us show first that q_* is a Cournot equilibrium. For $y \in [0, 1 - x_*]$ the expected profit of firm 2,

$U^2(x_*, y) = \frac{1}{3}y\left(1 - y^2\right) - \frac{2}{3}\frac{y}{100}$. has a (unique) maximum on $[0, 1 - x_*]$ at $y = y_* = {}^{3}/_{10}\sqrt{2}$. Thus, firm 2 has no incentive to deviate from y_* to another strategy in $[0, 1 - x_*]$. Now, for $y \in [1 – x_*, 1]$,

The maximum of $^{1}/_{3}y\ (1 - y^2) - {}^{2}/_{3}{}^{y}/_{100}$ on $[1 - x_*, 1]$ is attained at $y = {}^{7}/_{30}\sqrt{} \approx 0.571\ 55$. This maximum is equal to $^{343}/_{6750}\ 6\sqrt{} \approx 0.12447$, and therefore firm 2 has no incentive to deviate from y_* (that gives it a payoff $U^2\ (x_*, y_*) \approx 0.15274$) to a strategy in $[1 - x_*, 1]$. Since producing more than 1 would yield a negative expected profit, we have shown that firm 2 will not deviate unilaterally from q_*. By symmetry, the same holds for firm 1, and thus q_* is indeed a Cournot equilibrium.

$U^2(x_*, y) = \frac{1}{3}y\left(1 - y^2\right) - \frac{2}{3}\frac{y}{100}$. We show next that q_{**} is a Cournot equilibrium. For $y \in [1 - x_{**}, 1]$ the expected profit of firm 2,

$U^2(x_{**}, y) = \frac{1}{3}y\left(1 - \left(\frac{7}{30}\sqrt{6} + y\right)^2\right) + \frac{1}{3}y\left(1 - y^2\right) - \frac{2}{3}\frac{y}{100}$, reaches the maximum value of $^{343}/_{6750}\ 6\sqrt{} \approx 0.12447$ at $y = y_{**} = {}^{7}/_{30}\sqrt{6}$. Thus, firm 2 has no incentive to deviate from y_{**} to another strategy in $[1 - x_{**}, 1]$. For $y \in [0, 1 - x_{**}]$, the expected profit of firm 2 reaches the maximum value of ≈ 0.11798 at $y \approx 0.36792$. Hence firm 2 has no incentive to deviate from y_{**} to a strategy in $[0, 1 - x_{**}]$

Since producing more than 1 would yield negative expected profit, this shows that firm 2 will not deviate unilaterally from q_{**}.By symmetry, the same holds for firm 1, and thus is another Cournot equilibrium of the duopoly.

SUMMARY AND CONCLUSION

Throughout this paper we maintained the assumption that the set of states of nature Ω is finite. However, this assumption is by no means necessary, and was made only to simplify the presentation. In Einy et al (2007), a discussion paper on which this article is based, the uncertainty is represented by a probability space (Ω*, F, μ*), where Ω is a (possibly infinite) set of states of nature, *F* is a σ-field of subsets of Ω, and μ is a common prior. Firm *i*'s information is described by a σ-subfield F^i of *F*, which is not necessarily generated by a partition of Ω. The results on existence and uniqueness of I Cournot equilibrium remain valid in this more general context. Their proofs follow very closely those presented here, but some additional assumptions are made, which are not needed when Ω is finite. In particular, it is assumed that the demand intercept Q is bounded, and that the state-dependent inverse demand function, cost functions, and their first and second order derivatives, are bounded uniformly in w on some sufficiently big interval.

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