

COMPARATIVE STUDY OF OPTIMAL PORTFOLIO SELECTION USING ALGORITHM IMPERIALIST COMPETITIVE AND CULTURAL EVOLUTION

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Abstract

Due to the growing capital markets and the increasing volume of data, investors are looking for a quick solution to maximize efficiency and minimize risk to their formation of the optimal portfolios. Today, there are varieties of methods to optimize decisions and choices, these methods which are inspired by nature, such as intelligent search algorithms alongside traditional methods, have shown considerable success. The aim of the present study is selection of portfolio optimization algorithms based on Markowitz's mean-variance model and use of the colonial competition and cultural evolution algorithm for companies listed in Tehran Stock Exchange during the years 2007 to 2013 and comparing them with each other. The results show imperialist competitive algorithm has fast convergence to the cultural evolution algorithm and lead to a solution, even with a lot of calculations done in the least possible time and with the best results.

Keywords: Portfolio optimization, imperialist competitive algorithm, cultural evolution algorithm, mean semi – variance model

INTRODUCTION

At first glance, what may seem to the novice investors is that they invest the entire amount on companies that have maximum efficiency and by this way they could get most profitable income, but the biggest company's shares may decline. Allocation of funds to purchase one share will carry a high risk. Thus forming a portfolio of companies and diversification of the shares will lead to lower investment risk. Question is that which of the shares will have choose and put them in your cart and how much investment should be allocated to each of the shares? The purpose of diversification is not that high number of shares that make up the basket and keep up the great number of shares does not necessarily lead to a reduction in the risk portfolio that is made. But also to reduce the risk of the portfolio, those shares should be chosen that have less dependent to each other and to the extent possible, if one of them falling stocks, they should have a negative correlation with each other, until caused losses have offset by another profits of stock. (Shahr Abadi, 2010).

According to expert opinion, one of the reasons of underdevelopment of the developing countries is low levels of fixed investment in these countries. The most important issues of third world countries were absence of appropriate structure for individuals and organizations' money. On the other hand, the importance of active participation of investors in the stock market is so important that the nature of exchange existence is dependent on investment (Ramooz 2005).

The existence of an active and thriving capital market is always fitted as a sign of developed countries at the international level. In developing countries, most investments are done through financial markets. Active participation of the population is voucher in lives of the capital markets and sustainable development of the country. The major problem that investors faced in these markets is the decision to select securities for investment and the optimal stocks portfolio. Because of the complexity of the problem of optimal portfolio selection, algorithms, which have been proposed for the administrative standpoint is very time consuming and are not suitable for practical applications. In this study, the stock market will be presented to test competing colonial of innovative algorithms and cultural development to optimize the portfolio at the Tehran Stock Exchange.

LITERATURE REVIEW

Huiling Wu et al. (2014), in a study entitled "multi-period portfolio Markowitz based on mean-variance likely out of the state dependence ", examines the issues of possible chosen time horizon to calculate the portfolio based on multiple mean-variance. So that are assumed that the time horizon is selected by random and based on risks that have been determined by the market. This issue has been studied by studying the effective boundaries provided by an

analysis of the numbers, and independence possible time horizon (non-deterministic) to market standards, is proved.

Sefyani and Boziyani (2012), form five simple portfolios in a study entitled "Portfolio Selection Using Genetic Algorithm" and achieved remarkable results show that genetic algorithm has high efficiency and fast convergence which optimal solutions in the shortest possible time and with the best results.

Bermudez et al (2011) began to select the optimal portfolio in a study using genetic algorithms and fuzzy ranking. In this research, returns and portfolio risk of data exchange companies in Spain has been modeled by using fuzzy values and genetic algorithms and concluded that the use of genetic algorithm is an efficient method for portfolio selection.

Zhou et al. (2011), in a study entitled "portfolio optimization using particle swarm algorithm" has been modeled the algorithm to solve the problem of portfolio optimization and they concluded that the particle swarm algorithm with higher computational efficiency than the genetic algorithm, selects the optimal portfolio.

Anagnostopoulos (2010), in the research of portfolio selection problem, in addition to taking measures of risk and return, they considered minimizing the number of stocks in the portfolio as a third criterion. Also, they have been involved the main limitations and restrictions in the model category. The resulting optimization problem is a mixed-integer nonlinear three objective that to solve this problem, three kinds of evolutionary multi-objective optimization technique, is proposed. Their risk measure is also mean-variance risk measure.

Zare Mehrjouie and Rasaie (2013), in a study entitled, "Comparison of innovative methods for portfolio optimization under semi-variance risk criteria by using the t statistics test ", are considered information about the historical value of shares DAX, Hang Seng, S & P 100, in the period 2007 to 2008 as an input to the model. In this study, simulated annealing algorithm and prohibited search algorithm is used. The results show that the ability of these two algorithms in the semi-variance optimization model and consequently finding the efficient frontier is the same. In general, due to more quickly prohibited search algorithm to achieve optimal results, the use of this algorithm is proposed, to find efficient frontier under the semi-variance model.

Pour Kazemi et al (2013), in a study entitled "Optimizing portfolio of interaction with the Imperialist competitive algorithm (ICA)", are doing this work with the formulation of portfolio choice and regard to the interactions between the projects, and using ICA optimizing algorithm. The results show that ICA method compared to algorithms GA, PSO, CPSO that has been used in such matters, is superior.

Abbasi and Abovaly (2012), in a study entitled, "Optimal Portfolio Selection Using Genetic Algorithm NSGA-II", are used the data of the top 50 companies in Tehran Stock Exchange for the 2006-2010 years and they are considered the value at risk measure as a measure of risk. The results indicate that the multi-objective genetic algorithms can be used to select the optimum portfolio and portfolio performance designed by genetic algorithm is different with the top 50 companies with equal weights.

Sajjadi (2011), in a study entitled, "portfolio optimization using Imperialist competitive algorithm based on value at risk and its assessment" has participated in Tehran Stock Exchange. In this research, the purpose of an investment is to have maximum and minimum risk that risk measure used in this model is a value at the worth risk. Following the Imperialist competitive algorithm implementation and comparison with the genetic algorithm, the results show the significant superiority of the Imperialist competitive algorithm, meaning that the Imperialist competitive algorithm in addition to gain more profit, it spends less time to reach.

THE MEAN SEMI - VARIANCE

One of the most used models for portfolio selection is the model of Markowitz. He was the first one who extends formally the concept of diversification in portfolio. Markowitz model has data or inputs that are:

1. The expected return per share
2. The expected return standard deviation as a measure for determining the risk per share
3. Covariance, as a measure to show the alignment of the output portion.

Markowitz model is used of the two measures of return and risk associated with the investment budget constraints in the form of a second quadratic programming that was presented in 1952 with the name average - variance. According to the Markowitz model, optimal portfolio should optimize the risk in exchange for a maximum level of efficiency or the optimal portfolio maximizes the investment return for maximum acceptable level of investment risk.

Markowitz model was built based on the expected return and risk of securities and portfolio diversification that is essentially a theoretical framework for analyzing the risk and return options. According to his theory, efficient portfolio is a portfolio that has the highest return on a certain level of risk or the lowest risk for a given level of output. Markowitz had particular attention to its investment objective in formulating criteria "Risk Return". He did not search the investment risk only in SD, but he considered the effect of an investment on the risk of the investment risk (Eslami and Heibati, 2005).

Markowitz's mean- variance standard method attempts to intercept an efficient frontier to select a portfolio. This boundary is a continuous curve that shows exchange between return and portfolio risk. Briefly Markowitz's optimization model is presented as follows (Estrada, 2007):

$$\begin{aligned} \text{Min } Z &= \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \delta_{ij} \\ \text{subject to:} \\ \sum_{i=1}^n \omega_i \alpha_i &\geq R \\ \sum_{i=1}^n \omega_i &= 1 \\ \omega_i &\geq 0 \quad i=1,2,3,\dots,N, \quad \omega_j \geq 0 \quad j=1,2,3,\dots,N \end{aligned} \quad (1)$$

The result is called the efficient frontier when R is favorable investment returns and the model is solved for different number of R and results obtained from the objective function, that is in fact the risk, are drawn with equivalent number of R on a graph. In above model δ_{ij} shows stock covariance; i & j , ω_i and ω_j show stock weight, i & j , α_i show the average yield and i & R show a certain level of efficiency.

Due to the fact that in practice the efficiency of the portfolio is often asymmetrical distribution, in 1959, Markowitz offered a semi-variance model for asymmetric random returns. The researchers found that asymmetric returns turn out variance to an inefficient measure for risk (Estrada, 2007).

Using variance or the square root of the standard deviation as a measure of risk is difficult. These criteria for an asset that is normally distributed and traded in an efficient market, is an acceptable criterion. If these two features are not available for the assets, using the variance can be difficult. For this reason, other criteria are considered for the risk, which the mean-variance is among them (Abdolali Zadeh and Eshghi, 2003).

According to this criterion, only the random returns that are lower than average yields are used in the calculation of risk. In fact, in this risk definition deviation where the expected return deviation is dangerous that is losses to investor, otherwise, deviation of returns does not make any risks. Therefore, we replace zero with the difference between them in the calculating risk, when the random yield is higher than expected returns. (Jin et al, 2006).

With entering λ -coefficient in the objective function try to be included both measure risk and return in the objective function. While minimizing the risk is causing to maximize efficiency.

In fact, λ is only a weighting parameter that its value is changing in the range [0, 1] and the value of the investor to risk or returns is applied by itself. Le by increasing λ , increasing efficiency became important and at the same time as the value $(1 - \lambda)$ decreases, the weight of minimum target of risk become less. The proposed model can be rewritten as follows:

$$\text{Max } \lambda \sum_{i=1}^n \omega_i \mu_i - (1 - \lambda) \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \delta_{ij}$$

Subject to:

$$\begin{aligned} \sum_{i=1}^n \omega_i &= 1 \\ \omega_i &\geq 0 \quad i=1, 2, 3 \dots N \end{aligned} \tag{2}$$

In the above model, $\delta_{i,j}$ is j, i covariate, and ω_j the weight of stock j, μ_i indicates the average return i.

But the main limitation and disadvantage of this method is to inability to optimize the portfolio selection problem under the constraints of integer. Since in the real world and in the real financial decisions, most investors need to determine the exact number of assets in their portfolio, so entering an integer constraints, make the model closer to real world and therefore, the solution is offering practical and profitable decisions in the hands of investors.

Integer constraints are added to the model as follows:

$$\sum_{i=1}^n z_i = k \tag{3}$$

Based on these limitations, if we invest in i share, the amount of z_i is equal to one and if not invested in this share, the z_i value is equal to zero. In this formula, k is the number of shares that the investor is willing to have it and invest in his shopping cart. As can be seen, entering this limited converts a continuous search space to a discrete non-linear space, which rise to a complex combination of quadratic programming, and integer nonlinear, which is a difficult problem to solve. Finally the proposed model will be provided to the following to optimize the portfolio subject:

$$\text{Max } \lambda \sum_{i=1}^n \omega_i \mu_i - (1 - \lambda) \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \delta_{ij}$$

Subject to:

$$\sum_{i=1}^n \omega_i = 1 \tag{4}$$

$$\begin{aligned} \sum_{i=1}^n z_i &= k \\ \omega_i &\geq \cdot \quad i = 1, 2, 3, \dots, n \\ Z_i &\in \{\cdot, 1\} \end{aligned}$$

In this study, with the use of semi-variance instead of the variance in the above model, the new model is achieved that is called average- semi variance model which is offered in the following way (Gazkar, M., 2010):

$$\text{Max } \lambda \sum_{i=1}^n \omega_i \mu_i - (1 - \lambda) \sum_{i=1}^n \sum_{j=1}^n \omega_i \omega_j \text{ semi cov } i, j$$

Subject to:

$$\begin{aligned} \sum_{i=1}^n \omega_i &= 1 \\ \text{semi cov } i, j &= \frac{1}{T} [\sum_{t=1}^T \min\{(r_{i,t} - \bar{r}_i), \cdot\} * \sum_{t=1}^T \min\{(r_{j,t} - \bar{r}_j), \cdot\}] \end{aligned} \quad (5)$$

$$\begin{aligned} \sum_{i=1}^n z_i &= k \\ \omega_i &\geq \cdot \quad i = 1, 2, 3, \dots, n \\ Z_i &\in \{\cdot, 1\} \end{aligned}$$

$$t = 1, 2, 3, \dots, T$$

META-HEURISTIC ALGORITHMS

Imperialist competitive algorithm

Imperialist competitive algorithm is a technique in the field of evolutionary computation to find the optimal solution of the optimization problem deals. This algorithm offers algorithms for solving optimization mathematical problems with mathematical modeling process of socio-political evolution.

The main bases of this algorithm are formed of matching policy, competition colonial and revolutionary. This algorithm offers regular functions in the form of an algorithm that can help to solve complex optimization by imitating the evolutionary process of social, economic and political and with the mathematical modeling of the process. In fact, this algorithm is looking for the optimization problem answer in the countries and trying to improve this answer gradually during the iterative process.

Like other evolutionary algorithms, this algorithm begins with a random initial population that each of them called "country". Some of the best elements of the population (Equivalent elite in genetic algorithm) are selected as the imperialists. The remaining population is considered as a colony. Colonizers, depending on their power, attract these colonies with a special process. The total power of each empire depends on both its constituent parts, namely, the imperialists (the central core) and its colonies. In mathematics mood, this dependence is modeled with definition of imperial power, as the sum of the imperialists, plus a percentage of the average of its colonies.

With the formation of early empires, imperial rivalry begins between them. Each colonial empire, which cannot succeed in colonial compete and expand their power (or at least prevent its influence reduction), will be removed from scene of colonial competition. Therefore, the survival of the empire will depend on its ability to attract rival colonial empires, and bringing them to dominate. As a result, during the imperialist rivalries, strength will add gradually to larger empires and weaker empires will be removed. Empires will be forced to develop their own colonies to increase their power. Over time, the colonies will be closer to the empires in terms of power and we will be witnessing a convergence. The extreme of the colonial rivalry will happen when a single empire exists in the world that is very close to the imperialist countries themselves, with colonies of position (Atashpaz, 2008).

Cultural evolution algorithm

Cultural algorithm was proposed in 1994 by Reynolds. This algorithm is inspired by the evolution of human culture and the influence of a community and its impact on future generations. This algorithm uses the knowledge for the search process. Adding knowledge evolutionary is effective in improving the efficiency of evolutionary algorithms and makes the search process more intelligently. In fact, adding knowledge of mechanized is to reduce the scope of the search space by pruning the inappropriate part of it. This algorithm has different knowledge in its belief space that helps the search.

Cultural algorithm is a dual system inherited that offers two search space; one is space population, which is based on Darwin's theory of genetic and the other is belief space, which offers a part of culture, that, this case is distinction between genetic algorithm with cultural algorithm.

In fact, belief space models culture information of people. Population space offers individual in genotyping or phenotyping level. Both space work in parallel and influence on each other. To connect between these two spaces a communication protocol is defined. One of them

is to select a group of people to form a belief space and the other is a method to affect this belief space on people's producing in the population space.

Generally, the cultural algorithm works as: In every generation, first people arrive such as genetic algorithms in the population space and evaluated by a fitness function. Then, people are selected who are appropriate for forming belief space by the acceptance function and people's accepted practices are applied to make and change belief space (here, culture is simulated).

Created culture in belief space affected on the evolution of the population. This effect takes place with changing the mutation operator and its application in the production of children (Becerra, and Coello, 2006).

METHODOLOGY

In the present study to determine the sample, the specific relationship is not used to estimate sample size and sampling but due to the limited number of qualified firms, the removal method is applied. In other words, those population companies that have the following conditions are selected as sample and the others are deleted. To select a sample of the target population the following conditions are considered:

- Companies should be accepted until March 2005 in stock.
- Financial history companies should lead to the end of March each year.
- The company should not have to change their financial year during the periods.
- Companies should not have suspension during the periods.
- Companies should not have to be component of investment companies and their activity should not have to be generating.
- Annual equal volume of companies should not have to be less than 50 (on average one transaction per week).

Data required for this study, financial data of 106 different companies with both risk and return operating for 8 consecutive years since 2007 to 2013 which were collected by the Rahavard Novin application and various sites in Tehran Stock Exchange. To implement the algorithms, MATLAB tools, version 7.6 is used.

Basically importing raw data, reduce the speed and accuracy of the algorithm. To avoid this situation, and also to equal the value of the data, before the test, the input data must be normalized, it means that all data between 1 and -1 should be equivalent. In this study, data were normalized before the test and then the algorithm was studied by MATLAB.

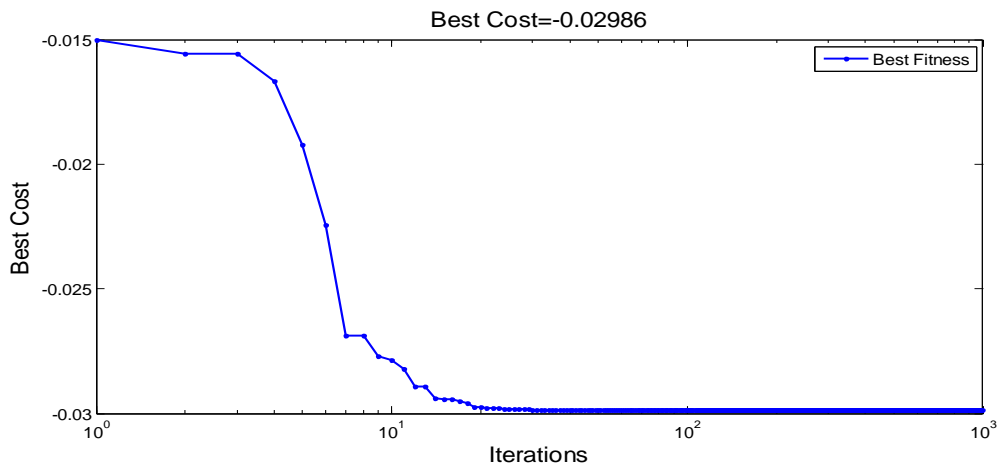
$$Y_i = \frac{y_i - y_{\min}}{y_{\max} - y_{\min}} (h_i - L_i) + L_i \tag{6}$$

Where Y_i is the input values normalized by the equation, y_i is the values of the input, y_{\min} is the smallest input value, y_{\max} is the largest amount of input (here + 1) = h_i is the high value in normalized distance and (here -1) = L_i is the bottom in normalized distance.

EMPIRICAL ESTIMATION

The results of the 1000 iteration of the algorithm implementation, caused to the appropriate convergence and Figure (1) shows path traveled by the evaluation function to reach the optimum point by imperialist competitive algorithm.

Figure 1. results of imperialist competitive algorithm



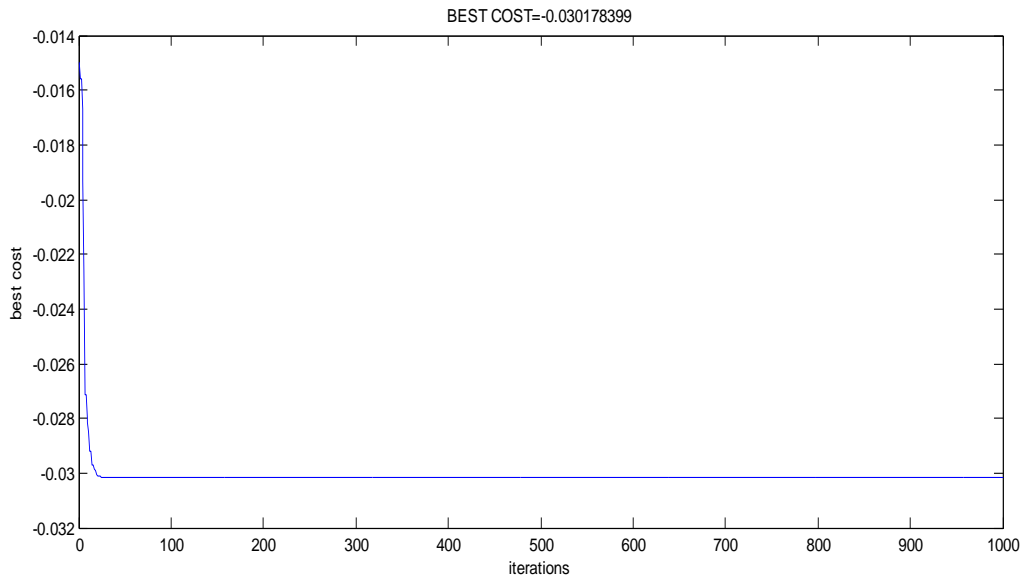
To test the stability of imperialist competitive algorithm, we repeat the algorithm 5 times to make sure after test, we obtained almost identical results. Variance 7 / 5152e-06 shows the stability of this algorithm.

Table 1. Study of the stability of colonial competitive algorithm in selecting the optimal portfolio

The objective function Run 1	The objective function Run 2	The objective function Run 3	The objective function Run 4	The objective function Run 5
0/039786	0/039856	0/03927	0/033845	0/038534
Average	0/028258			
Variance	7/5152e-06			
The best objective function value of 5 runs	0/039856			

The results of the 1000 iteration of the algorithm caused the appropriate convergence and Fig (2) shows path traveled by the evaluation function to reach the optimum point by cultural algorithm.

Figure 2. Path traveled by the evaluation function to reach the optimum point by cultural algorithm



To test the stability of imperialist competitive algorithm, we repeat the algorithm 5 times to make sure after test, we obtained almost identical results. Variance $2 / 97792e-06$ shows the top stability of this algorithm.

Table 2. Study of the stability of the cultural algorithm in selecting optimum portfolio

The objective function	The objective function	The objective function	The objective function	The objective function
Run 1	Run 2	Run 3	Run 4	Run 5
0/030178399	0/0299969	0/0301128	0/03002334	0/029988
Average	0/030062045			
Variance	2/97792e-06			
The best objective function value of 5 runs	0/030178399			

COMPARISON OF ALGORITHMS WITH DIFFERENT INPUT INFORMATION IN PORTFOLIO

Table (3) shows the results of the algorithm portfolios based on model of Markowitz. The results of the optimization aspects and the point of convergence rate (objective function) are comparable.

Table 3. The results of the algorithms portfolios comparison

Algorithm	Returns Cart	Risk Cart	The objective function
Imperialist competitive algorithm	27.047662	9.983187	00.039856
Cultural algorithm	24.935890	9.069320	0.030178399

Two following tables display the ranking results of algorithm selected portfolios based on semi-variance average model of Markowitz. Table (4) has been ranked in aspects of convergence and the table (5) has been ranked in aspects of the most optimal portfolio.

Table 4. ranking algorithms information portfolio regard to the return and risk Cart, based on the criteria Markowitz mean semi-variance aspects of convergence

Algorithm	The objective function	Ranking Algorithm
Imperialist competitive algorithm	00/039856	First
Cultural algorithm	0/030178	Second

Table 5. ranking algorithms information portfolio regard to the return and risk Cart, based on the criteria Markowitz mean semi-variance aspects of convergence

Algorithm	The objective function	Ranking Algorithm
Imperialist competitive algorithm	2/709321	First
Cultural algorithm	2/749477	Second

CONCLUSIONS AND RECOMMENDATIONS

In this study, we were followed to select the optimal portfolios with the use of annual return and risk firms. For this purpose, the sample included 106 companies were selected from the target population, including stock companies with a series of restrictions. In the next step we designed the research model; the research model was presented with entering the actual market constraints and preferences of different actions of investors based on Markowitz mean-semi-variance model. After designing the model, we design various algorithms related to the model and the model was run for each algorithm after several tests and determination of the algorithm parameters. In order to assess the stability and uniqueness optimum algorithm, we repeat

algorithm for 5 times; among the functions obtained, the best objective function is selected and we obtained the optimal portfolio for each algorithm by the objective function. Imperialist competitive algorithm gained first and cultural algorithm gained second rank of the optimality and imperialist competitive algorithm gained first and cultural algorithm gained second rank of the convergence.

In this model to assess the risk of assets, annual stock return has been used. Using monthly stock returns, increase the accuracy of the model. Moreover, the following is suggested to improve the designed model, which forms implications for the further research:

- In this paper only the information about the risks and returns of assets is used. In order to more efficient algorithms, we can also consider the liquidity of the securities.
- Design an algorithm taking into account transaction costs and taxes.
- Design an algorithm by considering the lower risk.
- The shares offered in the Stock Exchange are considered as existing assets in this algorithm.
- We can also consider a mix finance of stocks and other assets such as bank deposits, bonds, currencies, gold and real estate and implement the algorithm according to them.

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