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A MULTI-OBJECTIVE LINEAR PROGRAMMING MODEL FOR NATIONAL PLANNING

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Abstract

This article examines the applicability of multi-objective decision making methods in national planning. A multiple objective, project selection, linear programming model is developed to be used as a planning tool for a hypothetical economy. The model is solved using one of the important multiple objective linear programming methods developed by Benayoun et al. which is known as the step method or STEM, for short. Assumed logarithmic, additively separable utility functions, differing in their degree of nonlinearity have been developed and utilized to provide simulated decision maker's responses needed by the method. The article emphasizes that for national plans to be optimal, realistic, logical, and meaningful, multiple objectives mathematical programming models, based on "projects" rather than "sectors", are needed in formulating these plans.

Keywords: Multi-objective decision making, STEM, linear programming, national planning, utility function, optimization, project selection

INTRODUCTION

National planning and Mathematical Models

The growing ambition of developing countries to catch up economically with the developed industrial countries as rapidly as possible has been reflected in the acceptance of development planning as a pivotal means towards the attainment of growth and progress at the fastest pace. National planning is a complex process involving many different organizations and individual agents interacting in the formulation and execution of a country's economic and social policies. The planning boards in developing countries have been constantly engaged in the process of preparing comprehensive development plans in order to set forth, in a logical and consistent

manner the priorities, objectives, and aspirations of their countries. By planning their economies, they have hoped to avoid the apparent deficiencies of the market economy and at the same time mobilize additional resources, master existing resources efficiently, bring interdependencies and externalities into the decision making calculus, change expectations, and resolve inequalities, imbalances, and inconsistencies.

In practice, two general approaches have been used to coordinate the national plans, Morva (1975), the first is the traditional or conventional approach and the second is manifested in studies suggesting the use of mathematical models for this task. This work belongs to the second approach. By the means of mathematical models, the entire network of indirect and intricate interdependencies of the overall economic system can be traced and revealed. Mathematical models can provide a complete simultaneous solution to the planning problem. These models can be used to check the feasibility and consistency of a particular plan and to ensure its optimality (or efficiency) in the use of available information and limited resources, Qayum (1975).

The use of mathematical models, especially linear programming (LP), as a device for making the most efficient use of resources has made economists look with much favor at the significance of using these models in national development planning. Quite a few linear programming models have been put forward in the field of development planning, the important of which are those of Adelman and Sparrow (1966), Blyth and Crothall (1965), Bruno (1966), Chenery and Kretschmer (1956), Chenery and MacEwan (1966), Hjerppe (1976), Kornai (1967,1969), Kornai and Liptak (1965), Manne(1963,1966,1974), Morva (1970,1975), Nugent (1970), and Sandee (1960). Literature in this area makes use of some variations of the LP models with a' single' welfare objective function. Those models are strictly 'sectoral' models working with input-output ratios for different sectors. Our criticism of sectoral modeling is that, while the consistency framework of a multi-sector model highlights gross inter-relationships between sectors and it may guarantee consistency at the 'sectoral' level, it seems in many cases not to give directly operational information at the 'project' level within the sector. When the economy becomes more complex and the model thus becomes bigger, the identification of projects within a sectoral aggregate becomes very difficult and the relevance of multi-sector forecasts to project appraisal becomes too remote.

Multi-Objective Decision Making

Previous models have ignored 'multiplicity' of objectives and, in many cases, it is casually assumed that a well- established, well-identified and agreed upon single objective function is readily available for insertion into any model which happens to need it. Unfortunately, this is far

from the truth. In practice, planners are constantly faced with the fact that more than one objective function should be considered simultaneously if they seek to formulate a closer-to-reality plan. Certainly, in complex industrial or governmental problems, the decision maker (DM) may repeatedly have to select among alternative courses of action on the basis of 'multiple' and conflicting objectives.

This multiple and heterogeneous nature of objectives in such situations has called for an approach that can take this nature into consideration. Multi-objective decision making is an area of multiple criteria decision making that is concerned with mathematical optimization problems involving more than one objective function to be optimized simultaneously. Multi-objective optimization has been applied in many fields of economics, science, finance, engineering, and logistics where optimal decisions need to be taken in the presence of trade-offs between conflicting objectives. For a multi-objective optimization problem, there is no single solution exists that simultaneously optimizes each objective. In this case, the objective functions are said to be conflicting and there exist a number of Pareto optimal solutions. A solution is called non-dominated; Pareto optimal, Pareto- efficient or non-inferior, if none of the objective functions can be improved in value without degrading some of the other objective values. A multi-objective optimization problem can be formulated as

Max $(C^1(X),\ldots\ldots, C^k\ (X))$

Subject to $AX \leq b$ (1)

$X \geq 0$

Different researchers solve this problem in different ways. One approach is to convert the problem with multiple objectives into a single objective problem (a scalarized problem). However, when decision making is emphasized, the solution of the multi-objective optimization problem involves the decision maker (DM). The best solution to this problem is the one which is preferred most by the DM.

The most preferred solution can be found using different philosophies. Accordingly, the solution methods can be divided into 3 types: a priori, a posteriori, and interactive methods. All of them involve obtaining preference information from the DM in different ways. In a priori methods, preference information is first asked from the DM and the solution best satisfying these preferences is found. In a posteriori methods, a representative set of Pareto optimal solutions is first found and then the DM must choose one of them. In interactive methods, the DM is allowed to iteratively search for the most preferred solution. In each iteration of the interactive methods, the DM is shown a Pareto optimal solution(s) and describes how the solution(s) could be improved. The information given by the DM is then taken into account.

Many approaches have been put forward in the literature to deal with those methods of solution. These approaches have been tracked in a number of studies. The most comprehensive of them are Roy (1971), Cochrane and Zeleney (1973) , StarrandZeleney (1977), Zeleney (1973,1976,1982), Thiriez and Zionts (1976), Zionts (1978), Cohon and Marks (1975), Bell, keeney andRaiffa(1977), Farquahar (1984), Sfeir-Yonnes and Bromley (1977), White (1983,1984), Belenson and Kapur (1973), Jonhnson (1968),Chankong and Haimes (1983), Evans (1984), French (1983,1984), Ignizio (1982), ReVelle et al. (1981), Reitveld(1980), Hwang andMasud(1979), MacCrimmon (1973), Rothermel and Schilling (1984), Cohon (2004), Steuer (1985), Steuer and Na (2003), Weistroffer et al.(2005),and Miettinen et al. (2008).

One major stream of the suggested approaches to which this work is related, assumes that the final reconciliation between the conflicting objectives is a matter of value judgment that only the DM can make. In other words, it is asserted that the problem of conflict between objectives is due to the incomplete ordering of the objectives. Such incomplete ordering, which is a characteristic of a multi-objective optimization problem, signifies that without preference information, an optimal solution for this problem cannot be found since all feasible solutions are not ordered and thus not comparable, Cohon and Marks (1975), Cohon (2004). A complete ordering, which is a characteristic of a scalar single-objective optimization problem can be obtained for a multi-objective problem only by introducing value judgment in the solution process. With this in mind, this type of approach suggests that the principle of preference or utility is the answer. Bearing the principle of utility in mind, problem (1) above becomes as

$$\text{Max } U\,[C^1(X), \ldots, C^k(X)]$$
$$\text{Subject to } AX \leq b \quad (2)$$
$$X \geq 0$$

where U is the overall utility (preference) function defined on the values of the objectives, X is the constrained set of feasible decisions, and $C^1, \ldots, C^k$ are k distinct linear objective functions of the decision vector X. It is assumed that the DM has in mind, consciously or unconsciously the principle of utility when he ranks his objective. Thus, the adoption of this principle of utility can help resolving the conflicts inherited in a multiple-objective situation.

In such a situation, the final decision is one that maximizes the DM's utility and the concept of an ‘optimal’ solution is replaced by that of an 'efficient ' solution, Belenson and Kapur(1973), Geoffrion et al. (1972), or the best compromise solution, Belenson and Kapur (1973), Benayoun et.al. (1971) and Dyer (1973).

Each efficient solution implies values for each of the k objectives and the collection of all efficient solutions the set of efficient solutions. Without preference information, not one of the efficient solutions is preferable to any other efficient solution, but when preferences are known,

as represented by an indifference surface, for example, and then one of the efficient solutions can be identified as the "best-compromise" solution. In other words, the solution of the problem will be a point at which the efficient set of solutions and the indifference curves, which represent the contours of the DM's utility functions, are tangent of each other. Thus, the solution will have the highest utility for the DM, and it will also be efficient. Since *U* is not explicitly known, certain information is needed about it from the DM. This could be done either by uncovering the DM's overall utility function over the set of objectives or by inferring certain characteristics of this function from information provided by the DM. A number of methods for doing this have been put forward, Benayoun et al. (1971), Farquahar (1984), Geoffrion et al. (1972), Keeney and Raiffa (1976), Keeney (1981). One of these methods is the method of Benayuon et al. known as the step method or STEM for short (1971).In this article, we will solve our multi-objective model using this method and thus investigate how the method can be applied to national development planning.

The Method of Benayoun et al. (STEM)

One of the important methods that deals with multiple objectives is the step method, or STEM for short, described by Benayoun et al. (1971).STEM is an iterative exploration procedure, where the best compromise solution between the conflicting objectives is explored step by step towards the solution that will finally be chosen. In other words, the best compromise solution is reached after a certain number of cycles. Each cycle (m) consists of (1) a calculation phase and (2) a decision phase. The calculation phase consists of the construction of a "pay-off" table, and the computation of a feasible compromise solution through the minimization of the weighted sum of deviations from each individual optimum. The compromise solution is then proposed to the DM in the decision phase. The pay-off table is found by computing the optimum solution for each individual objective function using the following system:

$$\text{Max } C^j = \sum_{i=1}^{n} c_i^j x_i , \quad j = 1, \dots, k \qquad (3)$$

Subject to

$$Ax \leq b \text{ and } x \geq 0 \qquad (4)$$

Where C^j is the jth objective function to be optimized, c_i^j is the contribution vector for the ith decision variable for the jth objective, *A* is the vector of coefficients in the constraints, x_i is the vector of variables, *n* is the number of variables, and *k* is the number of objectives. Let *D* be the feasible region defined by constraints (4). In general, there is no feasible optimal solution for this multiple objective problem within the feasible region *D*. The problem is thus to find the solution *X* which will be the best compromise solution between the conflicting objectives. STEM is used to

find this best compromise solution. Before the first cycle starts, a pay-of table is constructed. For the feasible region *D*, defined by constraints (4), the optimum for each objective in turn is calculated and the pay-off table, Table (1) is made.

Table 1: Pay-off Table for STEM

The solution vector which optimizes the jth objective	The Value of jth objective					
	C^1	C^2	…	C^J	…	C^K
X^1	M^1	..	..	Z_j^1	..	..
X^2	..	M^2	..	Z_j^2	..	..
….	..	..	..	..	..	..
X^J	Z_1^j	Z_2^j	..	M^J	..	Z_k^j
….	..	..	..	..	..	..
X^K	..	..	..	Z_j^k	..	M^k

Row *j* in Table (1) corresponds to the solution vector x^j maximizing the objective function C^j under constraintsnumber (4). So z_i^j is the value taken on by the objective function C^i when C^j reaches its maximum M^J .

The main diagonal of this table (from top left to bottom right) contains the optimum values of the objectives and it represents an ideal solution x^*which is infeasible, i.e., the optimum values of the objectives (M^1, M^2,…, M^J, M^K).The problem is to find a 'compromise solution' X^m which is the nearest to the ideal solution X^*. The compromise solution is obtained solving the following LP problem:

Minλ

Subject to

$\lambda \geq [M^j - C^j(x)].\pi^j$, j=1,…,k (5)

$x \in D^m$

$\lambda \geq 0$

where λ is the weighted deviation of an objective from the ideal solution and it is to be minimized. The weights π_j are introduced to define the relative distances between the objectives and the ideal solution. Their determination is influenced by the values of the objectives in the pay-off table, and they are calculated as:

$\pi^j = \alpha^j / \sum_{j=1}^{k} \alpha^j$, $\sum \alpha^j = 1.0$ (6)

$\alpha^j = \frac{M^j - m^j}{M^j} \{\sum_{i=1}^{n} (c_i^j)^2\}^{-½}$ (7)

wherem^j is the smallest value which the jth objective takes in the jth column of Table (1). The weights π^j are normalized by equation (7) to make the different solutions obtained from different weighting strategies be easily compared. Thus, the π^j represent normalized weights on the various objectives which depend on the variation of the value of the objective from the optimum solution M^j.

In the decision phase, the ideal solution X^* and the compromise solution X^m are presented to the DM. Comparing them, he decides whether the compromise solution is satisfactory or otherwise. If it is satisfactory the compromise solution is the solution required and the procedures terminate. If it is not, the DM must accept a relaxation of a relatively satisfactory objective within the compromise solution to improve the values of the other objectives. He then indicates that objective and the maximum amount of relaxation he can accept. The problem is then modified in a manner which incorporates the DM's reactions and a return to the calculation phase is made. The new modified problem would look like

$$D^m$$

$$D^{m+1} C^{j*}(X) \geq C^{j*}(X^m) - \Delta C^{j*} \quad (8)$$

$$C^j(X) \geq C^j(X^m) \qquad , j \neq j^* \quad (9)$$

where$C^{j*}(X)$is the value of the satisfactory objective, and $\Delta C^{J*}$is the amount of relaxation in the satisfactory objective. Constraints (8) are additional constraints to the feasible region D^m. Their role is to limit the reduction in the value of the satisfactory objective to no more than the permissible reduction. Constraints (9) ensure that the unsatisfactory objectives values do not decrease. The weights π^j is then set to zero and the calculation phase of cycle m+1 begins.

The cycles continue until the DM is satisfied with the results, an indication that the best compromise solution is reached. If at any cycle the DM feels that not one of the objectives is satisfactory, then STEM stops with the conclusion that no best-compromise solution exists.

The authors of STEM have mentioned that the algorithm produces the best-compromise solution in fewer cycles than the number *k* of the objective functions involved and that fewer cycles are needed if the DM relaxes several objective functions at once, Benayoun et al. (1971, p. 372).

THE DESCRIPTION OF PROPOSED RESEARCH MODEL

The purpose of this article is to develop a plan in the form of a mathematical multi-objective LP model for a hypothetical economy that can be used in development planning and to solve this model using the method of Benayoun et al.(1971) known as STEM.

The Model's Projects

Our model is formulated in the framework of an assumed project selection situation. We assume that forty public projects have been assessed, appraised and found relevant to the developmental objectives. These projects cover most activities of the economy. They include agricultural, industrial, educational, social and regional projects.

The model gives a choice for each project to start in any year in the planning period. The idea of different starts for the projects is meant to give planners flexibility in shifting resources from one year of the planning period to another where they are optimally needed without disturbing the plan.

The Developmental Objectives

We assume that there are seven national objectives proposed for the plan (i. e. for the model). These objectives are:

1. The maximization of per capita income
2. The minimization of foreign aid used to fill the import-export gap
3. The maximization of skilled manpower
4. The minimization of foreign aid used to fill the investment-saving gap.
5. The minimization of the level of foreign technical expertise.
6. The maximization of the level of social services and infrastructure.
7. The maximization of the level of regional development.

It is worth noting that:

a. Maximizing the level of per capita income in objective no. 1 is done in the form of maximizing the net present value (NPV) of the per capita income of the projects, not only over the planning horizon, but also for the post-horizon data starting from the year that follows the plan up to the end of the project's expected economic life. The project's post-horizon data (its cash flows, either outlays or income) are discounted back, using a social discount rate, to obtain their NPV at the horizon, i.e., the year that follows the planning period. For instance, if the planning period is three years, there will be four values of per capita income for each project: one at the end of each of the three years of the planning period, and the fourth one is the summary figure of the post-horizon values discounted at

the end of year 4. All of these four values are discounted back to obtain their NPV now which is to be maximized. The NPV approach has also been used for objectives no. 2 and 4.

b. In order to compute the contributions of the model's projects to objective no. 5; we have used a scoring system on the scale from zero to ten, depending on the need of the projects to foreign technical know-how (or expertise).

c. The project that contributes to objectives nos. 6 and 7 is given a score of 1, whereas the project that does not, has a score of zero for its contribution to the relative objective.

The Model's Constraints

In recognition of the resource limitations of an economy, the requirements of the projects cannot exceed the available amounts of resources (either domestic or foreign). We assume that the basic restrictions of our hypothetical economy are on:1. local funds, 2. Foreign funds, 3. skilled manpower, 4. other resources: we have chosen three of the intermediate resources which can be critical to development. The shortages in any of them may hinder development. These resources are energy, cement, and steel. It has to be noted that there is a certain degree of interdependence between the projects of the model since we allow for the possibility that some project will benefit from the output of others. For example, the project "Electricity-generating Station" is expected to produce electricity in the third year of the planning period and this is included in the amount of energy available in the economy in that year (the RHS of the energy constraint). Of course, if the planning horizon were longer than it is assumed to be, more and more projects, for example, the Cement Factory project, the Electricity-generating Station and the Steel Factory project, would start producing such resources as cement, energy, steel, respectively, and their output would be included in the nationally available resources from which other projects may benefit.

The Plan Period

Our model can be used for any planning period provided that the required and relevant data are available. However, the planning period adopted in our model is three years (medium-term plan) for reasons of simplicity, and manageability. It is supposed that our medium-term plan is formulated within the framework of a more long-term perspective plan (say, twenty to twenty – five years). This is because the perspective plan can act as a guide to policy makers by exposing bottlenecks which will emerge as the economy expands if an anticipatory action is not taken well in advance. A medium-term plan, like ours, can also be supported by an annual plan. The annual plan in this case is used as a controlling plan in the sense that it is this which, year

by year, matches resources to possible achievements. It is guided by the medium- term plan, which sets its direction, but the annual plan is the operative document.

The Data

The data of this model are hypothetical but they are guided by the data published in the literature with respect to less developed countries. The data given in national planning models referred to in the introduction (Section 1.1), have also been useful. In addition to this, published plans for India, Sandee, (1960), and data books related to planning in Egypt, Ikram (1980), and Pakistan, Chenery and MacEwan (1966) have been beneficial.

To guarantee the necessary randomness in the data of our model we have used a computer model and a Monte Carlo simulation routine. This routine will yield random numbers, taken from a normal distribution with a certain mean and a certain standard deviation, if called with different initial values.

Different Versions of the Model

We have made three versions of our model (versions A, B, and C). These versions are mutually exclusive. The data for versions B, and C have been generated by a simple Monte Carlo program from the original version A. Having three versions of the model will allow us to experiment more than once with the application of STEM. It is possible to think of these versions as that the projects for each version are being available from different industrialized countries or as using different levels of technology, or as proposals from three different companies in response to invitation to tender).

MATHEMATICAL FORMULATION OF THE PROPOSED MODEL

The seven objectives set forth above can be mathematically expressed as

Max $c_1 = \sum_{i=1}^{p} \sum_{k=1}^{t} \gamma_{ik} x_{ik}$ (10)

Min $c_2 = \sum_{i=1}^{p} \sum_{k=1}^{t} \xi_{ik} x_{ik}$ (11)

Max $c_3 = \sum_{i=1}^{p} \sum_{k=1}^{t} \psi_{ik} x_{ik}$ (12)

Min $c_4 = \sum_{i=1}^{p} \sum_{k=1}^{t} \nu_{ik} x_{ik}$ (13)

Min $c_5 = \sum_{i=1}^{p} \sum_{k=1}^{t} \omega_{ik} x_{ik}$ (14)

Max $c_6 = \sum_{i=1}^{p} \sum_{k=1}^{t} \delta_{ik}\, x_{ik}$ (15)

Max $c_7 = \sum_{i=1}^{p} \sum_{k=1}^{t} \eta_{ik}\, x_{ik}$ (16)

The utility function to be maximized is

$U\,[c_1\,(x), c_2\,(x), \ldots, c_7(x)]$ (17)

Subject to

$\sum_{i=1}^{p} \sum_{k=1}^{t} \varphi_{ikr\Theta} x_{ik} \leq \beta_{r\Theta}$, r = 1, 2,…., *R*and Θ = 1, 2,…. , *t*(18)

$\sum_{k=1}^{t} x_{ik} \leq 1.0$,(19)

$0 \leq x_{ik} \leq 1.0$, for all *i*, and all *k* (20)

where

x_{ik}:The level undertaken of project *ik*.

γ_{ik}:The net present value of the contribution of project *ik* to the objective of“ maximization of percapita income".

k:The year in which project *i* starts.

i :The reference number of a project in each year *k*, where *i* = 1,…. , *p*.

t :The number of the final year of the planning period.

ξ_{ik} :The net present value of the contribution of project *ik*to the objective of “minimization of foreign aid needed to fill the import-export gap”.

ψ_{ik} :The contribution of project *ik* to the objective of ' maximization of skilled manpower'

ν_{ik} :The net present value of the contribution of project *ik* to the objective of “minimization of foreign aid needed to fill the investment-saving gap”.

ω_{ik} :The contribution of project *ik* to the objective of 'minimization of the level of technical expertise'.

δ_{ik} :The contribution of project *ik* to the objective of ‘maximization of the level of social services and infrastructure'.

η_{ik} :The contribution of project *ik* to the objective of ' maximization of the level of regional development'.

$\varphi_{ikr\Theta}$:The amount of resource *r* required by project *ik* in year Θ of the planningperiod.

$\beta_{r\Theta}$: The amount of national resource *r* available for the projects of the current year of the plan in year *Θ*

R:The number of all resources considered in the model.

c_1 - c_7:The seven linear objective functions of the model.

U :The utility function to be maximized.

Constraint (19) constrains the project from being taken more than once during the planning period. The objective functions and the constraints of the model are assumed to be linear. It is important to emphasize that although any model should be a reflection of reality, no model is a perfect reflection. This is so for many reasons. A model is an abstraction which can only incorporate certain aspects of the real world. Many economic relationships cannot yet (if ever) be formulated either in quantitative or qualitative terms. Besides, all models leave out relationships and details which could, in principle, be included.

ASSESSING THE DM'S UTILITY FUNCTION

Previous applications of multi-objective methods were founded on actual experiences of a single DM (or a group of DMs) who has (or have) an implicit utility function(s). The DM would supply the information needed by the method regarding his preferences. This paper, however, has a different orientation. It takes the approach of simulating the DM's responses, needed by the method of Benayoun et al. (STEM)(1971) using a hypothetical explicitly known utility function as a substitute for the DM. The use of this function makes it possible to solve our model described in Section 3 above. Using a utility function for the DM ensures the most satisfactory solution to the DM. The solution will be a point at which the non-dominated set of solutions and the indifference curves of the DM is tangent to each other (the indifference curves can be considered as contours of equal utility). Thus, the solution will have the highest utility for the DM and it will also be non-dominated. Although there are different forms of aggregate utility function, Hwang and Masud (1979), Keeney and Raiffa (1976), Keeney (1981), and Keelin (1981),we have assumed that our DM's utility function is additively separable with respect to the objectives considered. This assumption permits the consideration of each of the objectives independently and the assessment of a single dimensional utility function defined on each. It also implies the existence of *p* single utility functions $u_1, \ldots, u_p$ such that

$U(C) = \sum_{i=1}^{p} u_i(c_i)$ for $C = (c_1, \ldots, c_p)$ (21)

where *U (C)* is the aggregate utility of the DM related to objective vector *C*.

As for the single utility functions, it is assumed that they are logarithmic functions of the form

$u_i = \Lambda \log_{10}(\Xi + c_i) + \Gamma$ i = 1, 2,….., p(22)

where Λ, Ξ, and Γ are constants, and c_i is the value of each objective i.

The single utility, u_i, is then normalized, i.e. consistently scaled between 0.0 and 1.0 such that it is 0.0 when the value of the corresponding objective is minimum and it is 1.0 when the value is maximum. Both the minimum and maximum values for each objective considered depend on the project data of the model. To find the minimum and maximum values for each objective separately, we have solved our model using each of the objectives independently as an objective function for the model which is solved once as a minimization problem to obtain the minimum value, c_m, of the objective, and another time as a maximization problem to obtain the maximum value of the same objective, c_x. Thus, the normalized value of this objective, c_{iz}, at the current point is calculated as:

$$c_{iz} = \frac{c_i - c_m}{c_x - c_m} \quad (23)$$

where c_i is the current value of objective i obtained from the multiple-objective model, and c_m and c_x are both the minimumand maximum values, respectively, that could be obtained for the same objective. Thus, equation (22) becomes as equation (24)

$u_{iz} = \Lambda \log_{10}(\Xi + c_{iz}) + \Gamma$, i = 1,2,….., p(24)

Through this normalization process, all objectives and utilities, irrespective of their different wide-ranging values, can be easily viewed and compared. The logarithmic form of the single-dimensional utility function has been chosen because it is plausible to suppose that a utility increases with the corresponding objective but at a diminishing rate. A function having this property is the logarithmic function.

Furthermore, we have divided the non-linear curvature of the utility function (24) into three different forms, each with a different degree of non-linearity. Since non-linearity itself differs along the curvature, it is worth investigating utility in its different degrees of non-linearity. The difference between the three forms of non-linearity is in the values of the constants Λ,Ξ, and Γused in the single dimensional utility functions. These constants ensure that the logarithms in the single utility functions are defined throughout the range values which the objectives can assume.

Accordingly, we have used three forms of utility functions for our DM to correspond to the three degrees of non-linearity after experimenting with different coefficients (constants).These forms are:

1. The almost linear form: $u_{iz} = 24.16 \log_{10}(10.0 + c_{iz}) - 24.16$ (25)

2. The ordinary non-linear form: u_{iz} = 3.322 $\log_{10}$ (1.0 + c_{iz}) (26) 3. The highly non-linear form: u_{iz} = 0.5$\log_{10}$ (0.01 + c_{iz}) + 1.0 (27)

where u_{iz} and c_{iz} are the normalized values of the utility functions and the objective functions, respectively.The classification of our utility into different types of non-linearity is based on the power series expansion of Taylor's theorem. Accordingly, equation (21) becomes as equation (28).

U_z (C) = $\sum_{i=1}^{p} u_{iz}$ (c_{iz}) , for C = (c_{iz}, ….,c_{pz}) (28)

where U_z the normalized is aggregate utility and u_{iz} is the normalized i^{th} single-dimensional utility.

THE APPLICATION

For solving our model using STEM, we start the calculation phase by constructing the pay-off tables for the three versions of our model as the method itself explains; Benayoun et al. (1971). These pay-off tables (Tables 2, 3, and 4) are constructed after calculating the optimum for each objective in turn of our model given in section 3 above in its 3 versions. In each pay-off table, row *j* gives the values of the seven objectives for the solution X^J related to each objective. The main diagonal of the table contains the optimum values of the objectives (shown in bold and italic) and it represents the ideal solution.

Tables (2), (3), and (4) show the pay-off tables for versions "A", "B", and “C”, respectively.

Table 2: The pay-off table for version “A” of the model

The solution vector which optimizes the jth objective	The Value of jth objective						
	C^1	C^2	C^3	C^4	C^5	C^6	C^7
X^1	***5.14***	- 596.34	38731.29	-41.76	25.83	1.00	0.00
X^2	-0.99	***16.42***	1851.67	1.15	1.00	1.00	0.00
X^3	-21.64	-1139.64	**147247.73**	-79.78	55.45	8.00	8.00
X^4	-0.99	16.42	1851.67	***1.15***	1.00	1.00	0.00
X^5	-20.68	-677.19	107668.88	-47.42	***120.52***	12.32	11.32
X^6	-21.74	-773.81	98122.56	-54.16	32.39	***18.00***	8.00
X^7	- 13.57	-452.99	56135.85	-31.70	31.00	8.00	***16.00***

Table 3: The pay-off table for version "B" of the model

The solution vector which optimizes the jth objective	The Value of jth objective						
	C^1	C^2	C^3	C^4	C^5	C^6	C^7
X^1	***9.34***	-1046.52	56587.62	-73.25	25.02	3.00	1.00
X^2	-1.78	***39.27***	1857.68	2.75	0.00	1.00	0.00
X^3	-18.35	-1167.46	***147152.47***	-81.72	49.32	9.99	8.00
X^4	-1.78	39.27	1857.68	***2.75***	0.00	1.00	0.00
X^5	-16.88	-567.67	81503.83	-39.72	***118.85***	10.93	10.93
X^6	-13.65	-1057.20	98548.36	-74.01	32.00	***18.00***	8.00
X^7	-7.78	-638.40	51431.99	-44.68	31.00	8.00	***16.00***

Table 4: The pay-off table for version "C" of the model

The solution vector which optimizes the jth objective	The Value of jth objective						
	C^1	C^2	C^3	C^4	C^5	C^6	C^7
X^1	***6.67***	-1040.17	70826.84	-72.80	21.00	1.00	0.00
X^2	-1.46	***34.32***	1391.13	2.40	1.00	1.00	000
X^3	-19.60	-1168.03	***139838.80***	-81.78	55.68	10.95	11.58
X^4	-1.46	34.32	1391.13	***2.40***	1.00	1.00	0.00
X^5	-19.96	-655.44	103107.82	-45.89	***118.30***	13.00	12.00
X^6	-18.91	-830.09	98747.10	-58.09	32.39	***18.00***	8.00
X^7	-6.70	-587.19	63530.40	-41.10	31.00	8.00	***16.00***

To find the first compromise solution X^m using the system of equations (5) above, we need to calculate the weights π^j. They are calculated using equation (6) and normalized using equation (7) and their total adds up to 1.0. This means that different solutions obtained from different weighting strategies can be easily compared. For our model the calculated initial π^j vectors for the three versions of the model are as shown in the following Table (5).

Table 5: The initial weighting vectors π^j for the three versions of our model

π^1	π^2	π^3	π^4	π^5	π^6	π^7	$\sum_1^7 \pi^j$
Version "A"							
0.1835	0.0399	0.000006	0.5816	0.0146	0.0840	0.0964	1.00
Version "B"							
0.1835	0.0303	0.000009	0.4314	0.0269	0.1527	0.1752	1.00
Version "C"							
0.2773	0.0270	0.000009	0.3858	0.0228	0.1378	0.1493	1.00

The weights (coefficients) π^j are then used in the system of equations no. (5) to find the first compromise solution. The values of the objective functions after this compromise solution as well as those of the ideal solution for our model are shown in Table (6).

Table 6: The first compromise solutions compared with the ideal solutions (non- normalized)

Version "A"							
solution	C^1	C^2	C^3	C^4	C^5	C^6	C^7
1st compromise (x^m)	-1.5317	-13.4780	9017.8905	-0.9550	36.6679	3.4257	4.1382
Ideal (x^*)	5.14	16.42	147247.73	1. 15	120.52	18.00	16.00
Version "B"							
1st compromise (x^m)	-1.3941	-25.8586	13031.8317	-1.8155	45.5286	5.1031	6.00
Ideal (x^*)	9.34	39.27	147152.47	2.75	118.85	18.00	16.00
Version "C"							
1st compromise (x^m)	-0.5159	-39.4573	4543.6892	-2.7500	30.9362	3.5445	3.3812
Ideal (x^*)	6.67	34.32	139838.8	2.4	118.30	18.00	16.00

According to STEM, both solutions in Table (6) are to be presented to the DM in the decision phase of this method. The role of the DM is confined to dealing with the following two points:

(1) to choose the most satisfactory objective from the compromise solution, and

(2) to relax this objective by a certain amount so that other unsatisfactory objectives can be improved.

To simulate the DM's role in taking a decision regarding these two points, we have introduced the utility approach into STEM (originally STEM does not deal with the utility theory directly). Using the utility approach (equations from 25 to 28 above), we can test the DM's reaction to the above two points. This has been done as will now be described.

We have solved several LP problems with feasible regions D^m corresponding to several rates of relaxation, such that $0.0 < \Delta C_1^{j*} < \Delta C_2^{j*} < \ldots < \Delta C_M^{j*}$ where C^{j*} is the most satisfactory objective obtained so far; ΔC_n^{j*}, n=1,…, M is the rate of relaxation in that objective, and ΔC_M^{j*} is the maximum acceptable rate of relaxation. For each LP problem, the feasible region is modified as in the system of equations from (8) and (9) mentioned above.

For each of the LP solutions corresponding to each ΔC^{j*}, we calculate the total utility of the solution using equations (25) to (28). The final 'best compromise' solution is the one that has the highest total utility for the DM. In practice, the DM is supposed to decide which one (or more) of the objectives is to be relaxed after the first compromise solution. But since we do not have our DM present we have relaxed all of the seven objectives by different rates of relaxation. By doing this, we have better insight into the behavior of the objectives so that we would know how each one of them responds to different rates of relaxation after the first compromise

solution. Even in practice, if the DM is present but not able to make up his mind about which one of the objectives is to be relaxed, relaxing all of them may help the situation.

Theoretically, we could carry on iterating for more cycles to reach the final solution while modifying the feasible region each time to include all previous cycles, but to consider all of our seven objectives and all the relaxation rates we have suggested, with different combinations, would mean solving a very huge and awesome problem. Therefore, we suffice ourselves with stopping after the second phase of STEM, as shown in Table (7).

As for the relaxation process itself, a rate of 10% from the value of the satisfactory objective seems a reasonable rate to start with, since it is neither unnecessarily small, nor unnecessarily large. The rate is then increased by another 10%, and so on until the relaxation rate reaches 50%. Thus, the relaxation rates used are 10%, 20%, 30%, 40% and 50%. We think that 50% is the maximum acceptable rate of relaxation because if an objective was allowed to be relaxed by more than 50% this would mean that this particular objective was not of sufficient relative importance to be one of the objectives considered, and it should not have been one of them in the first place.

Hence, for each version of our model, several LP's, corresponding to the above rates of relaxation, are solved. The calculation of the total utilities associated with them is done using equations (25) to (28) as has been done above. Even in practice, if the DM was present then, having seen the different values of the total utility corresponding to each rate of relaxation ΔC^{j*}, for all of the objectives, he can choose the best compromise solution, i.e., the one that gives him the highest possible total utility.

THE RESULTS OF APPLICATION

The results of these series of relaxation (performed on all of the seven objectives after the first compromise solution) are presented in Table (7), where the normalized total utilities for the 3 forms of non-linearity, corresponding to each rate of relaxation and for each version of the model, are projected. The values of the utilities obtained from the first compromise solution are shown in italic while the highest values of these utilities after relaxation of objectives are shown in bold.

I. for Version "A" of the model

Table (7) shows that the best compromise solution, for this version, can be obtained when either objective1, 2, or4 is relaxed. It also seems that relaxing these three objectives by any of the relaxation rates ranging from 10% to 50% would bring about the same value for the normalized total utility of the DM in its three forms of nonlinearity. The values of the utilities in the best

compromise solution are 3.8398, 4.1013, and 5.6152 for the almost-linear, ordinary and highly nonlinear utilities, respectively. The improvement on the first compromise solution is about + 5.2% (from 3.6498 to 3.8398), + 5.13% (from3.9010 to 4.1013) and 2.1% (from 5.5002 to 5.6152) for the three forms of nonlinearity, respectively. The relaxation of the other objectives (3, 5, 6, and 7) does not improve on the value of the total utilities obtained in the first compromise solution. It needs noting that since the relaxation rates from 10% to 50% would bring about the same values for the utility functions we prefer to choose the lower rate of relaxation which is 10%. It would be better to relax (or to worsen) the objective function by the smallest possible rate of relaxation.

II. for Version "B" of the model

Table (7) shows that the relaxation of objectives 1, 3, and 4 will improve on the value of the normalized total utilities obtained in the first compromise solution. The biggest improvement is obtained when objective 1 is relaxed by 10%. In this case we have the following values for the three forms of nonlinearity: 4.0723, 4.3790 and 5.8552, respectively. In percentage terms, the improvement is about + 5.4% (from 3.8634 to 4.0723), + 4.97% (from 4.1718 to 4.3790), and +1.78 % (from 5.7526 to 5.8552), respectively. The relaxation of the other objectives (namely 2, 5, 6, and 7) does not bring about any improvement on the values of the normalized utilities obtained in the first compromise solution.

III. for Version "C" of the model

Table (7) presents the results of relaxing the objectives of version "C" of the model. From this table, it can be seen that the relaxation of objectives 1, 2 and 4 increases the values of the normalized total utilities. The biggest increase, however, occurs when objective 1 is relaxed by either 40% or 50%. In this case, the values of utilities will be 3.7594, 4.0263 and 5.4979 for the three forms of nonlinear utility function, respectively. In percentage terms, this improvement from the first compromise solution is about + 8.22% (from 3.4737 to 3.7594), + 8.26% (from 3.7191 to 4.0263) and 3.6% (from 5.3069 to 5.4979), respectively. The relaxation of objectives 3, 5, 6, and 7 does not improve on the value of the total utilities. Thus, it can be said that the best

improvements in the three utility functions occur when objective function no.1 is relaxed by 40 % (being a smaller rate of relaxation than 50%). In Table 7 the normalized values of the total utilities, for the 3 versions of the model, shown against different rates of relaxation. The values of the utilities obtained from the first compromise solution are shown in italic, and the highest values of the utilities, after relaxation of objectives, are shown in bold.

Table 7: Final Results

		Version A of the Model			Version B of the Model			Version C of the Model		
		The Normalized values of the total Utilities								
		Almost Linear	Ordinary Non-Linear	Highly Non-Linear	Almost Linear	Ordinary Non-Linear	Highly Non-Linear	Almost Linear	Ordinary Non-Linear	Highly Non-Linear
		The Values of the Utility Functions after the 1st compromise Solutions								
		3.6498	*3.9010*	*5.5002*	*3.8634*	*4.1718*	*5.7526*	*3.4737*	*3.7191*	*5.3069*
The Objective To be Relaxed	The Rate of Relaxation	The Values of the Utility Functions Corresponding to Different Rates of Relaxation								
Objective 1	$\Delta C_1 = 10\%$	**3.8398**	**4.1013**	**5.6152**	**4.0723**	**4.3790**	**5.8552**	3.6903	3.9493	5.4567
	$\Delta C_2 = 20\%$	3.8398	4.1013	5.6152	4.0313	4.3258	5.8046	3.6903	3.9493	5.4567
	$\Delta C_3 = 30\%$	3.8398	4.1013	5.6152	4.0638	4.3708	5.8046	3.6903	3.9493	5.4567
	$\Delta C_4 = 40\%$	3.8398	4.1013	5.6152	4.0638	4.3708	5.8046	**3.7594**	**4.0263**	**5.4979**
	$\Delta C_5 = 50\%$	3.8398	4.1013	5.6152	4.0638	4.3708	5.8046	3.7594	4.0263	5.4979
Objective 2	$\Delta C_1 = 10\%$	3.8398	4.1013	5.6152	3.8634	4.1718	5.7526	3.7275	3.9898	5.4710
	$\Delta C_2 = 20\%$	3.8398	4.1013	5.6152	3.8634	4.1718	5.7526	3.7275	3.9898	5.4710
	$\Delta C_3 = 30\%$	3.8398	4.1013	5.6152	3.8634	4.1718	5.7526	3.7275	3.9898	5.4710
	$\Delta C_4 = 40\%$	3.8398	4.1013	5.6152	3.8634	4.1718	5.7526	3.7275	3.9898	5.4710
	$\Delta C_5 = 50\%$	3.8398	4.1013	5.6152	3.8634	4.1718	5.7526	3.7275	3.9898	5.4710
Objective 3	$\Delta C_1 = 10\%$	3.6498	3.9010	5.5002	3.9264	4.2360	5.7852	3.4737	3.7191	5.3069
	$\Delta C_2 = 20\%$	3.6498	3.9010	5.5002	3.9264	4.2360	5.7852	3.4737	3.7191	5.3069
	$\Delta C_3 = 30\%$	3.6498	3.9010	5.5002	3.9264	4.2360	5.7852	3.4737	3.7191	5.3069
	$\Delta C_4 = 40\%$	3.6498	3.9010	5.5002	3.9264	4.2360	5.7852	3.4737	3.7191	5.3069
	$\Delta C_5 = 50\%$	3.6498	3.9010	5.5002	3.9264	4.2360	5.7852	3.4737	3.7191	5.3069
Objective 4	$\Delta C_1 = 10\%$	3.8398	4.1013	5.6152	4.0513	4.2360	5.8387	3.6646	3.9267	5.4401
	$\Delta C_2 = 20\%$	3.8398	4.1013	5.6152	4.0513	4.2360	5.8387	3.6646	3.9267	5.4401
	$\Delta C_3 = 30\%$	3.8398	4.1013	5.6152	4.0513	4.2360	5.8387	3.6646	3.9267	5.4401
	$\Delta C_4 = 40\%$	3.8398	4.1013	5.6152	4.0513	4.2360	5.8387	3.6646	3.9267	5.4401
	$\Delta C_5 = 50\%$	3.8398	4.1013	5.6152	4.0513	4.2360	5.8387	3.6646	3.9267	5.4401
Objective 5	$\Delta C_1 = 10\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_2 = 20\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_3 = 30\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_4 = 40\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_5 = 50\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
Objective 6	$\Delta C_1 = 10\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_2 = 20\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_3 = 30\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_4 = 40\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_5 = 50\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
Objective 7	$\Delta C_1 = 10\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_2 = 20\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_3 = 30\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_4 = 40\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069
	$\Delta C_5 = 50\%$	3.6498	3.9010	5.5002	3.8634	4.1718	5.7526	3.4737	3.7191	5.3069

CONCLUSIONS

We have developed a multi-objective linear programming model which can be used as a tool for national development planning instead of the single-objective models which have been used by many researchers. Our model is based on projects rather than sectors. We have experimented with our model by solving it using one of the important and computationally efficient multi-

objective methods; that is the method of Benayon et al. known as the step method or STEM. This method requires the presence and interaction of the decision maker (DM) in order to reach to a compromise solution between the competing objectives. However, in our model we have developed an additively separable utility function, with different degrees of non-linearity, to replace the DM. This function would provide the necessary responses of the DM needed by STEM. The reasons for replacing the DM by the utility function are because of some of the difficulties involved in applying STEM since although STEM is computationally efficient as has been mentioned, it asserts that a best-compromise solution does not exist if the DM is not satisfied after a number of cycles equal the number of objective functions involved. This implies that the DM is not willing to reduce (or to worsen) the value of the satisfactory objectives to improve the unsatisfactory ones. STEM proposes that when this situation exists, then no decision can be made. This, of course, is not realistic as decisions in these situations must be made whether or not the DM is satisfied. Further, one of the assumptions of STEM is that the DM will be consistent enough to be able to specify, at a given point, which objective function he wishes to relax and by how much. In many practical complex situations, this may not be the case and the above assumption may not be realized. In this case, the production of an acceptable solution could be a long process. Furthermore, in STEM, it is not possible to improve the objective which had previously been worsened. This would lead to a non-desirable solution. Thus the DM could not change his mind during the solution process, also in using STEM; some difficulty was experienced by the subjects of the experiments in specifying the maximal amount of relaxation of the objective at each cycle, Wallenius and Zionts (1976).To overcome these difficulties, we have developed and used our utility function in the solution process. We have also experimented with STEM using 3 versions of our model and we relaxed all of the objective functions systematically to produce different solutions, where the solution that produces the highest value for the utility function is considered the best compromise solution. Even if the DM is present, the use of the utility function approach and the relaxation of all objectives with a series of relaxation rates, like what we have done above, can help the DM in interacting better with the method of solution.

Finally, from experimenting with our multi-objective model, we can say that the essence of improving the planning process is the incorporation of multiple objective methods; planning models are better off if the issue of multiplicity of objectives is adhered to and if policy makers (as decision makers) are given an active role to play in the solution process of a multiple-objective method used in planning with the assistance of the analysts who apply the method and the model.

SUGGESTIONS FOR FUTURE RESEARCH

Based on the findings of this article, several paths for further research can be suggested. The most important of these are:

1. The consideration of multiple or group decision making. Often, systems which are best modeled by multiple objective functions are also characterized by more than one decision maker.

The decision maker, if an individual, as opposed to a group, seldom makes a decision in vacuum. He or she is influenced by others; and in many instances groups, rather than individuals, make decisions. Therefore, what is needed is the development of a methodology for approaching the group decision making problem.

2. Apply multi-objective methods to decentralized planning with conflicting, multiple objectives. We believe that the introduction of multiple objective decision making, with its man-machine interactive approach, into the decentralization procedures of multi-level planning will facilitate the implementation and hence the applicability of these procedures. This belief has to be substantiated by more research and investigation. In a way, this direction of research could be related to the one mentioned above, since multi-level planning with multiple objectives involves various decision makers searching for a compromise of their individual objectives.

3. In the development of our model we have assumed the decision maker aggregate utility function to be additively separable. More research is needed to investigate different forms of utility function like multiplicative, quasi-additive, etc.

4. We have also assumed that the single utility functions related to the objectives to be logarithmic. Further research can investigate the use of other forms like, for example, exponential, quadratic, etc.

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