

**MEASURING INDEX VALUE - AT - RISK USING LAG OPTIMIZATION WITH STRESSED SCENARIOS**

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**Abstract**

*This paper attempts to discover the lagged optimization of two major Indian stock Indices using econometric assumptions. The main focus is to demonstrate the difference in the observed values of the Index-VaR (Index-Value at risk). Also, the Index VaR will further be tested using Stressed values. GARCH (Generalized Autoregressive Conditional Heteroscedasticity Model) and EGARCH (Exponential Generalized Autoregressive Conditional Heteroscedasticity Model) models were utilized for VaR estimation. For non-linear optimization the use of "Evolutionary Algorithm" using spreadsheet solver tool was used. Both the indexes are studied independently hence the preposition is under univariate modeling. The aim is to identify the difference in the risk optimization and the change in the Lags when an artificial or 'stressed simulated' environment is fitted within the actual historical price moves. To observe the lag movements and determination of optimal parameters in both GARCH (Generalized Autoregressive Conditional Heteroscedasticity Model) and EGARCH (Exponential Generalized Autoregressive Conditional Heteroscedasticity Model) environment, the traditional MLE-AIC (Maximum Likelihood Estimate-Akaike information criterion) minimax criterion was utilized. The result clearly demonstrated a marked difference in the VaR at various stress levels confirming that within Indices the 'Information arbitrage' is possible even at the monthly price movements.*

*Key Words: Portfolio Optimization, Value-at-risk, Lagged variable, GARCH, EGARCH*

**INTRODUCTION**

The literature mainly speaks off that on monthly basis lag of first order is correct or should be used for financial time-series. But, the present modeling framework is examining what happens if we simulate the prices on higher volatilities and thus sees the impact on the risks in form of VaR (Value-at-risk) of Index prices with the concept of AIC (Akaike information criterion) lag optimization. Now, administrating the index price risks using stylized stress scenarios is known in the academic literature, specially pertaining to the financial econometrics. The present paper is thus a mere extension of utilizing the stress scenarios with the Lag optimization essentially by

using AIC framework. And, with the stress or shock scenarios what are the changes in the values of Individual VaR by using GARCH (Generalized Autoregressive Conditional Heteroscedasticity) and EGARCH (Exponential Generalized Autoregressive Conditional Heteroscedasticity) models.

In India, BSE (Bombay stock exchange, India) and NSE (National stock exchange, India) are two major Indices, and their movements are considered proxies of Indian economic and capital formation. Hence, with this paper one can easily analyse that whether playing with Index movements (whether for trading, or for risk management) is essential and how does monthly lag values reflect the information content. If there is a change in the lag values across two indices and these changes are not similar then there is possibility of information arbitrage and investors can consider a pure portfolio of Index for investment purposes.

## LITERATURE REVIEW

Boonvorachote (2004) utilized the Trivariate Structured Vector Autoregressive Method (SVAR) method where the return, volatility and volume variables were used with optimal lags for each. The paper says that with regard to volume and return (mixture of distribution hypothesis) the information reaches together so the investor decides his/her buy/sell decision since information reaction is simultaneous. On the contrary, the sequential arrival of information model says that information of price volatility and volume reaction in groups and the decision as lead-lag framework. For checking the rate of information flow the number of transactions (changes in volume) can be a good proxy for speed of flow of information.

Guitierrez, Souza, and Gullien (2007) explained one of the methods of selecting the optimal lag order by using alternative criteria. Which selects simultaneously the lag order  $p$  and the rank structure  $S$  due to the WF restriction?

Fayad, Bates & Hoeffler (2012) used the AIC, BIC for their empirical study. The maximum of three lags were used in the time series dimension. The paper used the Lipset's law by taking the historical data of income in terms of GDP per capita of sample of 105 countries and polity IV Democracy Index. As per Lipset's Law, the democracy and growth in countries with no time variation were excluded because they remained either pure democracies or pure non-democracies for long. Lag optimization was used by HQ criteria. The result clearly speaks that countries with High resources usually show very low or negative relationships between Income and Democracy exhibit positive relationship.

Winker and Maringer (2004) explained that while using Multivariate linear (VAR, SVAR and VECM) and non-linear system of equations (MS-VAR), AIC, and SBC tests for optimal lag selection generally do not work when there is non-stationary and cointegration in time-series is selected. The authors applied two specific methods of optimal lag selection; one is Modified

Information Criterion (MIC) in the case of partially non-stationary VAR model. Another, is on reduced rank regressors were used. Another interesting concept of using “holes” i.e. Lags were not used in sequence from say Lag 1 to Lag N instead they were selected dynamically. In order to apply this dynamic model, Threshold Algorithm (TA) were used which was further amplified by Memeic Algorithm (MC).

Ali and Mustafa (nod), in their paper used the ADF test to check the Unit root existence among variables. For appropriate Lag length again the use of BIC and HQ were used. The second step was to check the cointegration among the variables. So that in case there exist any stationary then whether these non-stationary variables also reflect co-integration. For this Johanson Cointegration test with Trace statistics and Maximum Eugen value for test statistic were utilized.

Brandt, Santa-Clara and Volkanov (2007) interestingly used optimal portfolio weights as criteria for choosing the maximization of returns. The authors claimed that traditional methodologies of Markowitz values require a tedious task of involving returns and their covariance's for optimization purposes. But the paper only restricted to first order or one period lag for optimization purposes. So this “Lag function” was not there in the objective function.

## METHODOLOGY

For the present paper, the 191 months Index prices of Mumbai stock exchange (BSE) and National Stock Exchange (NSE) from India spanning from year 1996-97 till 2011-12 respectively. For estimation of Value-at-risk following techniques were used the detail of each of them is as followed:

### GARCH model

Generalized Autoregressive Conditional Heteroskedacity model usually referred as GARCH depicts the ARCH type model with conditional volatility attached. ARCH allows only lagged parameter to work, while GARCH works with one additional conditional lagged parameter. That is why, in the present paper, the GARCH lagged optimization is also utilized.

$$\sigma_{x,n}^2 = \omega v + \beta \sigma_{x,(n-k)}^2 + \alpha \mu_{(n-k)}^2 \quad (\text{Equation 1})$$

$\omega$  = long term weight

$v$  = long term volatility (191 months)

$\beta$  = it the parameter attached with the Lagged variance

$\alpha$  = it is the parameter attached with the lagged squared return

In terms of estimating the lagged parameters i.e.  $\omega$ ,  $\beta$  and  $\alpha$ , the use of Maximum Likelihood model (MLP) is used,

$$MLF = -\log \sigma^2 - \frac{\mu^2}{\sigma} \text{ (where } \omega, \alpha, \beta \geq 0, \text{ non-negative)} \quad (\text{Equation 2})$$

$-\log \sigma^2$  = this is the log of variance

$$\frac{\mu^2}{\sigma}$$

$-\frac{\mu^2}{\sigma}$  = This is also considered as Sharpe factor, since the return is divided by risk.

For MLP, the excel solver is utilized for calculation purposes.

### EGARCH model

Usually the limitation of the GARCH model is that it can show the “Leveraging effect” but cannot accommodate the asymmetries with the leveraging effect. For this reason, a better model called Exponential GARCH is also employed in the present model: the formula and its description are as follows:

$$\ln \sigma_x^2 = \omega_x + \beta \ln \sigma_x^2 + \gamma \left( \frac{\mu_{x(n-k)}}{\sqrt{\sigma_x^2(n-k)}} \right) + \alpha \left[ \frac{|\mu_{x(n-k)}|}{\sqrt{\sigma_x^2(n-k)}} - \frac{2}{\pi} \right] \quad (\text{Equation 3})$$

$\sigma_x^2$  Is the conditional variance for the one period ahead of any past information on the standardized residual?

$\beta, \omega, \gamma, \sigma$  are the parameters estimated using maximum likelihood method as described earlier

$\sigma$  Parameter represents the symmetry effect of the model or what we called as GARCH effect

$\beta$  Represents the persistence in the conditional volatility immune to any activity in the market larger  $\beta$  means that mean reversion is slow, or volatility takes larger time to smoothen out.

$(n-k)$  Represents the lag, in EGARCH (1,1) the lag is usually one period, but we can take multiple lag distribution as well. In the present model, for lag optimization, for monthly time frequencies, the lags from 1 to 5 months are kept.

$\gamma$  Parameter represents the asymmetry in leverage effect.  $\gamma = 0$  means model is symmetric,  $\gamma < 0$  means positive shocks (good news) bring less volatility than the negative shocks and if  $\gamma > 0$ , then it means that positive innovations create more volatilities than the negative events.

### INDIVIDUAL VaR 191 Month equation

For value-at-risk calculation the following formula for both the index series was used:

$$\text{VaR}_{\text{MonthlyStressed}} = -(\mu_x + Z\sigma_x\sqrt{n})P \quad (\text{Equation 4})$$

$\mu_x$  = the return of x series in the nth time period

$Z$  = the Z score based on the confidence interval, higher confidence interval leads to higher Z score, and indeed higher VaR value

$\sigma_x$  = The standard deviation of the x variable in the nth time period

$\sqrt{n}$  = the factor of frequency, for converting daily values to monthly this can be used. However, in present model this is kept as 1.

$P$  = the initial capital employed for trading, one must remember that the present model is using "Dirty stress testing" hence, the value of capital is kept constant throughout the 191 months' time horizon.

For stress series the following equation was employed:

With changes in

With stylized stress values subject to

$$= \text{if}((\mu_{(n-k)} \geq \mu_s, \mu_s, \text{if}(\mu_{(n-k)} \leq \mu_s, \mu_s, \mu_{(n-k)})) \quad (\text{Equation 5})$$

Where SV is the stylized stress percentage.

$\mu_{(n-k)}$  = this is the value of lagged return

$\mu_s$  = this is the value of stressed return, (artificially stimulated return, in the present paper 5%, 10%, 15% and 20% monthly stressed ranges were deployed)

For lagged Optimization, the Akaike information criterion (AIC) is a measure of the relative quality of a statistical model, for a given set of data. As such, AIC provides a means for model selection. So in the present model, instead of running the AIC function for the set of (lag m, lag n) combinations, the optimum lags at which the AIC will be minimum is determined.

The formula of AIC is as followed:

$$AIC = 2k - 2\text{Log}L, \text{ here}$$

$k$  = total parameters

$\text{Log}L$  = Maximum Likelihood estimate

Hence in Spreadsheet, to generate optimized results, two steps were employed:

Maximizing the MLP value for each index model

Minimizing the ALC value after the MLP is maximized using Evolutionary algorithm

## ANALYSIS & FINDINGS

The Table below presents the AIC lag optimization and estimates of Index VaR's.

TABLE 1: AIC lag optimization and estimates of Index VaR's with GARCH & EGARCH model implementation

Initial capital	50000 INR													
Stress testing	Model= Dirty (no change in the capital)													
Horizon	191 Months													
VARIABLES UNDER STUDY														
Confidence Level	99%													
Lag range	1-5 months													
<b>INDEX VaR using Lagged Optimization (Monthly)</b>														
<b>RISK ESTIMATE</b>	<b>GARCH optimization</b>						<b>EGARCH OPTIMIZATION</b>							
	$\alpha$	$\beta$	AIC	Optimal Lag 1	Optimal Lag 2	VaR (In INR)	$\alpha$	$\beta$	$\gamma$	AIC	Optimal Lag 1	Optimal Lag 2	VaR (In INR)	
Stress 5%														
BSE	0.1299	0	-588.35	4.08 (4)	4.99 (5)	6805.07	0.0115	0.00004	0	-554.33	3.35(3)	1.78(2)	8134.11	
NSE	0.078	0	-574.69	3.81 (4)	4.74(5)	6998.21	0.0034	0	0.0065	-553.49	2.93 (3)	1.95 (2)	8069.16	
Stress 10%														
BSE	0.3933	0.0181	-439.43	4.95 (5)	1.95(2)	10005.28	0.0067	0.001	0.0112	-427.54	3.35 (3)	1.78(2)	10159.31	
NSE	0.495	0	-419.65	2.06 (2)	4.52 (5)	10560.55	0.0034	0	0.0065	-409.78	2.61 (3)	1	11158.38	
Stress 15%														
BSE	0.323	0.1671	-395.44	4.95(5)	1.95(2)	11251.46	0.0081	0.0023	0.012	-376.73	3.35 (3)	1.78(2)	11937.22	
NSE	0.233	0	-367.77	2.06(2)	4.52(5)	12016.1	0.0034	0	0.0065	-362.63	1.47(1)	1	12168.66	
Stress 20%														
BSE	0.3525	0.1751	-374.52	4.95(5)	1.95(2)	11916.46	0.0138	0.0009	0.0277	-368.73	1.82(2)	4.72(5)	11646.38	
NSE	0.32	0	-337.68	2.06(2)	4.52(5)	12991.78	0.0034	0	0.0065	-335.24	1.47(1)	1	12676.24	

The time period taken in this paper is crucial, as it accommodates the major financial crisis happened during the 2007-08, plus host of internal reforms to boost the capital markets within country. So essentially, by all means the BSE and NSE being the principal indices should reflect the same results under the same simulated environment.

*With respect to LAG movements:*

Very interesting results are observed, since the result were only tested on Lagged components, once can observe that the reaction of BSE and NSE were different for lagged for symmetry and lagged for persistence, in GARCH, while at 5% stress levels, BSE were getting their optimize lags at 4.08 (4) and 4.99(5) respectively, NSE were observed at 3.81 (4) and 4.74 (5). Clearly, showing that at lower stress simulation, the NSE was more reactive (information absorbing faster) compared to BSE. while, at 10% there dynamics shifted, for NSE while for lagged value for index return improved to 4.95 (5), the lagged on variance (persistence) reduced to 1.95 (2), which means at 10% monthly range, the BSE had reached to its to persistence faster, meaning the mean reversion will be slow compared to NSE. While, from 10% till 20% no change in the Lagged positions were observed between the NSE and BSE respectively. Coming to the EGARCH component, there are two significant lag changes one appeared from 5% to 10% and

another from 15% to 20% change in the stressed values. it is worth appreciating, that BSE lags were insensitive from 5% to 15% , as they were 3.35 (3) for lag 1 and 1.78 (2) for lag 2, while, for NSE from 10% till 20% there was no change in the lag 2 it remained 1 (1) from 10% to 15% respectively, which means that 'persistence' remained the same. At NSE, lag1 remained constant from 15% to 20% at 1.47 (1) lags respectively. In EGARCH model, at 5% stress range, the BSE for lag 1 was higher compared to lag 2 (this was not the case with GARCH model as in that case, BSE was found lagging for both parameters). One also see that with respect to lag 1, from 5% to 10% shift, the NSE lag 1, reacted significantly faster in GARCH compared to EGARCH (technically the lag difference in NSE from 5% to 10% stress level was of 2 lags, while in EGARCH NSE lag 1 changes from mere 2.93 (3) to 2.6 (3) respectively).

With respect to EGARCH and in relation to "Persistence" i.e. lag attached to the return, it is worth analyzing that NSE the reaction to the stress shock was by a large much faster and remained persistent from 10% till 20%. (In GARCH, for lag 2, NSE moved from 4.74 (5) lags to mere 4.52 (5) lags and then these lags were stabilized, but at EGARCH for lag 2, NSE moved from 1.95 (2) and stabilized at 1 and remained there from 10% to 20%).

So from empirical perspective, one can essentially say that information arbitrage on the component of index (the stocks out of indices are made) greatly define the movement of the index, and lag optimization prove that it is possible that indices may react differently at different shock levels in future. Higher the rate at which the persistence is reached, higher the chances that index will absorb the shock and the market stabilizes fast. Lesser the rate of persistence with the increase in the shock component, the chances are that Market gets swayed away leaving investors in doldrums. By observing the above results, it has been seen that NSE reacted better at lag 2 levels compared to BSE.

This paper essentially speaks about 'Information arbitrage' since it can be seen that at various stress levels (alternatively shock levels) it is possible that index react differently to different news. So there will be an information lag, like we see in GARCH optimization that BSE even at monthly basis of 5% stress level, it was lagging in both components i.e. persistence and symmetry (GARCH) component in comparison to NSE. This difference further increase as the model shifted from 5% to 10% stress levels as we can see that optimal lag for BSE were at 4.95 (5) and 1.95(2) while that of NSE were at 2.06 (2) and 4.52 (5).

*With respect to Risk sensitivities:*

*GARCH optimization*

After observing the behavior of two indexes with lag optimization scenarios at various shock levels, the next pertinent task is to see how this reflected in the individual Risk values in form of

value-at-risk. Since, the VaR is measured in absolute terms, with respect to the Dirty stress testing approach, (the initial capital was kept at 50000 INR). For BSE and NSE, the most observable change in VaR profiles was from shift of 5% to 10% shock levels. While for BSE from 6805.07 INR it changed to 10005.30 INR. For NSE, this was respectively found moving from 6998.20 INR to 10561 INR. Technically, both BSE and NSE were seen having highest adjustments from 5% to 10% levels. It was 47.03% for BSE and 51% for NSE this change in VaR OF BSE and NSE slowed down to 12.5% and 13.8% from 10% to 15% shock movements which further goes down to 5.9% and 8.1% at 20% shock levels for BSE and NSE VaR values respectively.

#### *EGARCH optimization:*

The VaR responded slightly differently in EGARCH model, while from 5% to 10% change of stress component, the VaR of BSE increased to 24.8%, the NSE by the same change took the value to 38.3%. Similarly, from 10% to 15%, while the BSE changes to 17.5% in VaR value compared to this NSE reduced to mere 9%. (Although, both were found with positive change in VaR). From further shock i.e. 15% to 20% the VaR change at BSE was surprisingly -2.4% while for NSE it was still positive 4.2%.

Thus by observing the GARCH and EGARCH components, we can easily infer that EGARCH stands much better since it provide a more magnified view of risk movements across two indices with change in shock levels and at the lag optimization parameters. It is also closely observed through the above analysis that movement of NSE VaR values was slower compared to BSE, stating that information absorbing power and adjustment to the higher shocks is must faster in NSE compared to that of BSE.

## **CONCLUSION & MANAGERIAL IMPLICATION**

Risk sensitivity to shocks is an interesting phenomenon, and with use of higher level econometric tools, not only one gets a better view of risk, but also one can see how in reality indexes react to information shocks. And also what is the rate of absorption when these shock or stress levels are altered. So, lag optimization helps in identifying the rate at which information is absorbed and prices are adjusted into the trading assets in the stock markets.

From managerial point of view, stock exchanges in India should utilize such 'Shock rating scales' to observe how each index reacts to the shock components. Only those indices, whose rate of shock persistence improve with the shock levels must be allowed to run in the stock market.



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